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PREDICTION OF AERODYNAMIC ADMITTANCE FUNCTION USING FLUTTER DERIVATIVES

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Abstract

We propose a new prediction method of aerodynamic admittance functions for lift and moment of bridge decks utilizing flutter derivatives and these functions obtained by our method have been verified to be accurate.

key words : aerodynamic admittance, flutter derivatives, gust, identification, Sears’ function

Introduction

This paper describes new estimation method of aerodynamic admittance functions for lift and moment of buff body structures. Sears’ function is well known was a theoretical one of the aerodynamic admittance for lift of airfoils. Aerodynamic admittance functions for bridge deck sections (i.e., bluff bodies) have been reported to be considerably different from Sears’ function. A few empirical formulas have been proposed by Matsumoto (1975) etc. The formulas are considered applicable to limited types of deck sections which they gave. Therefore, the authors proposed a general method for estimating aerodynamic admittance function using flutter derivatives (i.e., FDs), and we compared the results with measured values obtained in actively generated turbulent flows.

Prediction of aerodynamic admittance function using flutter derivatives

Extension of Sears’ function

The lift and moment acting on an airfoil, flying at a uniform speed U and entering a sinusoidal gust with amplitude W, can be given in Equation (1) by Fung (1969).

$$\begin{aligned} &L = \pi\rho c U W e^{i\omega t}\varphi(k) \;\;,\;\; M_{1/2} = L\cdot {}^{c}\!/_{4} \\ &\varphi(k) = \left[J_0(k) - iJ_1(k)\right]\cdot C(k) + iJ_1(k) \end{aligned} \qquad (1)$$

where ρ : air density, c : wing chord length, $\phi(k)$: aerodynamic admittance function (Sears' function), $C(k)$: Theodorsen's function, $J_n(k)$: Bessel functions ($n=0,1$), k ($=b\omega/U$): reduced frequency, b : half chord length, and ω : natural circular frequency.

For the theoretical aerodynamic force given by Equation(1), the unsteady aerodynamic theory for a thin airfoil subjected to forced oscillation due to a periodic excitation was applied. The relative velocity on the airfoil varies according to whether the airfoil experiences harmonic oscillation or is subjected to a sinusoidal gust. In either case, however, the circulation lift is determined via Theodorsen's function. Therefore, the authors thought that Equation(1) might be applicable to bridge deck sections by replacing Theodorsen's function $C(k)$ for the airfoil with the equivalent Theodorsen's function $C_{eq}(k)$ (Scanlan(1974)) for a bridge deck section. Since the lift of bridge deck sections was expected to act in different way according to their shapes, the equivalent Theodorsen's function should be individually defined for lift and moment. The steady state derivatives of lift and moment coefficients were given by $dC_F/d\alpha$ and $dC_M/d\alpha$. Therefore Equation(1) was extended as shown below. In the equation, the wing chord length c is replaced by the bridge deck width B.

$$\left.\begin{aligned} L &= \frac{1}{2}\rho U^2 B \frac{dC_F}{d\alpha}\left(\frac{W}{U}\right)e^{i\omega t}\,\varphi_L(k) \\ M &= \frac{1}{2}\rho U^2 B^2 \frac{dC_M}{d\alpha}\left(\frac{W}{U}\right)e^{i\omega t}\,\varphi_M(k) \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned} \varphi_L(k) &= [\,J_0(k)-i\,J_1(k)]\,C_{eq,L}(k)+i\,J_1(k) \\ \varphi_M(k) &= [\,J_0(k)-i\,J_1(k)]\,C_{eq,M}(k)+i\,J_1(k) \end{aligned}\right\} \tag{3}$$

where, $\phi_L(k)$ and $\phi_M(k)$: equivalent Sears' functions defined for lift and moment, and $C_{eq,L}(k)$, $C_{eq,M}(k)$: equivalent Theodorsen's functions defined for lift and moment.

Identification of equivalent Theodorsen's function and Wagner's function

As shown in reference 3), the relationship between equivalent Wagner's function $\Phi_{eq,i}(\tau)(i=L,M)$ and equivalent Theodorsen's function $C_{eq,i}(k)$ is defined as follows.

$$\Phi_{eq,L}(\tau) = 1-c_1 e^{-c_2\tau}-c_3 e^{-c_4\tau} \tag{4-1}$$

$$C_{eq,L}(k) = F_L(k)+i\,G_L(k) \tag{4-2}$$

$$\mathrm{F}_L(k) = 1-\frac{c_1 k^2}{k^2+c_2^2}-\frac{c_3 k^2}{k^2+c_4^2} \tag{4-3}$$

$$\mathrm{G}_L(k) = -\left(\frac{c_1 c_2 k}{k^2+c_2^2} + \frac{c_3 c_4 k}{k^2+c_4^2}\right) \tag{4-4}$$

$$\Phi_{eq,M}(\tau) = 1 - d_1 e^{-d_2\tau} - d_3 e^{-d_4\tau} \tag{5-1}$$

$$C_{eq,M}(k) = F_M(k) + i\, G_M(k) \tag{5-2}$$

$$F_M(k) = 1 - \frac{d_1 k^2}{k^2+d_2^2} - \frac{d_3 k^2}{k^2+d_4^2} \tag{5-3}$$

$$G_M(k) = -\left(\frac{d_1 d_2 k}{k^2+d_2^2} + \frac{d_3 d_4 k}{k^2+d_4^2}\right) \tag{5-4}$$

where, c1～c4 & d1～d4 : unknown parameters and $F_i(k)$, $G_i(k)$ (i=L , M) : real and imaginary parts of equivalent Theodorsen's function. The relationship between equivalent Theodorsen's function and unsteady aerodynamic forces during harmonic oscillation is given by the following equations.

$$\mathrm{L}_h = -\pi\rho b^2 U a - \rho b U \frac{dC_F}{d\alpha} \left\{ F_L\left(U\alpha + \dot{h} + \frac{b}{2}\dot{\alpha}\right) + G_L\left(\frac{U}{\omega}\dot{\alpha} - \omega h - \frac{b\omega}{2}\alpha\right)\right\} \tag{6-1}$$

$$M_\alpha = -\frac{1}{2}\pi\rho b^3 U\dot{\alpha} + 2\rho\rho^2 U \frac{dC_M}{d\alpha} \left\{ F_M\left(U\alpha + \dot{h} + \frac{b}{2}\dot{\alpha}\right) + G_M\left(\frac{U}{\omega}\dot{\alpha} - \omega h - \frac{b\omega}{2}\alpha\right)\right\} \tag{6-2}$$

In this study, unsteady aerodynamic forces were measured by a forced oscillation device, and given by FDs H_i^* and A_i^* according to the notation of unsteady aerodynamic forces proposed by Scanlan (1974) et al. Then, the relationship between H_i^* (or A_i^*), and F_L, G_L (or F_M, G_M) was given by the following equations.

$$K^2 H_1^* = -\frac{dC_F}{d\alpha} K F_L$$

$$K^2 H_2^* = -\frac{K}{2}\left[\pi + \frac{dC_F}{d\alpha}\frac{F_L}{2} + 2\frac{dC_F}{d\alpha}\frac{G_L}{K}\right]$$

$$K^2 H_3^* = -\frac{1}{2}\frac{dC_F}{d\alpha}\left[2F_L - \frac{G_L K}{2}\right]$$

$$K^2 H_4^* = \frac{dC_F}{d\alpha} KG_L \qquad (7\text{-}1\sim4)$$

$$K^2 A_1^* = \frac{dC_M}{d\alpha} KF_M$$

$$K^2 A_2^* = -\left[\frac{\pi}{8} K - \frac{dC_M}{d\alpha} \frac{F_M}{4} K - \frac{dC_M}{d\alpha} G_M \right]$$

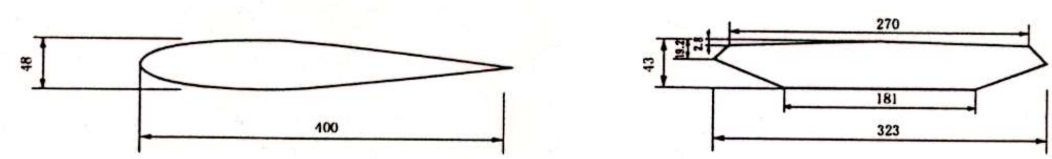
$$K^2 A_3^* = \frac{dC_M}{d\alpha} \left[F_M - \frac{KG_M}{4} \right]$$

$$K^2 A_4^* = -\frac{dC_M}{d\alpha} KG_M \qquad (8\text{-}1\sim4)$$

where $K(= B\omega/U)$: reduced frequency. Based on Equations (4)～(8) , it is obvious that H_i^* and A_i^* is expressed by a nonlinear function of parameters c_1～c_4 & d_1～d_4. In this study, not only these parameters but also the steady state aerodynamic force coefficients were identified so that the result might agree with measured values of H_i^* and A_i^*. The steady state aerodynamic force coefficients were identified as the unknown parameter c_5 & d_5. For parameter identification, extended Kalman filter (i.e., EK-WGI method) by Hoshiya (1984) was applied.

Comparison between measurement and prediction of aero-admittance function

In this study, an airfoil section and a flat box deck section without railing and curbs were used. Fig.1 shows deck section models.



(a) NACA0012 Airfoil (b) Flat Box

Unit:mm

Fig.1 Section Models for Wind Tunnel Tests

Aerodynamic forces due to wind gust were measured in a turbulent flow generated by an active gust generator. The active gust generator using arrays of airfoils and plates was applied to simulate the turbulence of Von Kármán's spectrum as target spectrum (target values: I_u=10%, I_w=5%, L_u=150cm, L_w=75cm) in the wind tunnel. Fig.2 shows the target and the measured power spectrum of wind gust. The measured power spectrum agreed well with the target ones. In this study, aerodynamic admittance function was also estimated directly from power spectrum of turbulent flows and aerodynamic forces. The direct estimated aerodynamic admittance functions were compared with indirect identified ones from FDs.

Equivalent Wagner's functions were identified from FDs. Fig.3 shows equivalent Wagner's functions for lift and moment of airfoil and flat box girder deck sections. The figure also shows the approximation of the Wagner's function given by R. T. Jones.

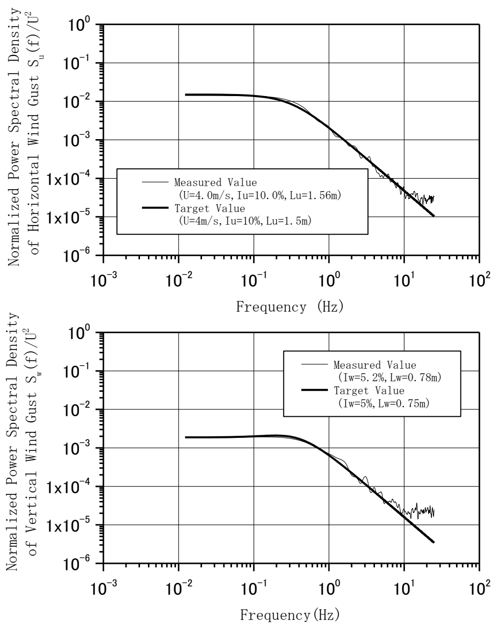


Fig.2 Measured and Target Power Spectrum of Wind Gust

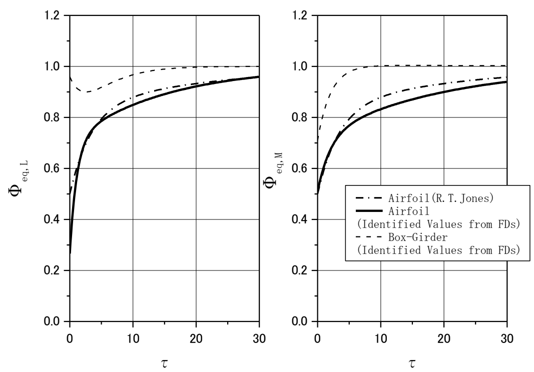


Fig.3 Measured and Identified Values of Equivalent Wagner's Function

These values are almost in good agreement with the theoretical ones for airfoil. The values for the flat box deck section tend to be dissimilar to the theoretical ones. Equivalent Wagner's function of flat box deck section has the identical tendency with values of a flat plate section given by Yoshimura & Nakamura (1975) as shown in Fig.4.

Fig.5 and Fig.6 respectively show direct and indirect estimated aerodynamic admittance functions of the NACA0012 airfoil and flat box deck sections. For the airfoil section, both measured and identified aerodynamic admittance functions are in good agreement with Sears' function. However, some discrepancies in high frequency region may result from the periodicity of Bessel function.

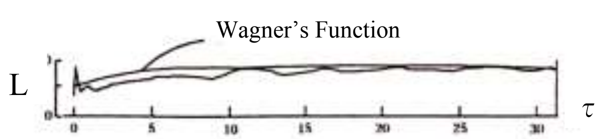


Fig.4 Measurement of equivalent Wagner's function of flat plate section by Yoshimura & Nakamura6)

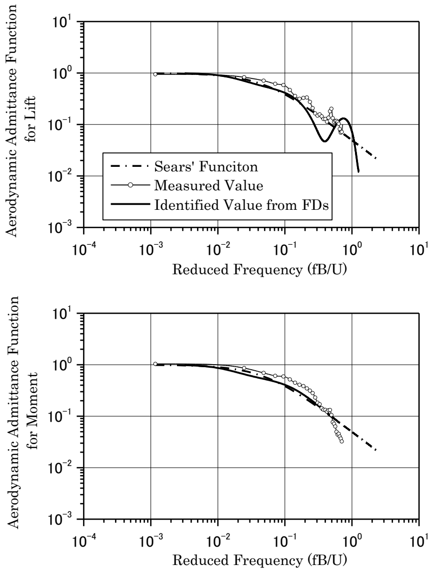


Fig.5 Measured and Identified Aerodynamic Admittance functions of NACA0012 Airfoil Section

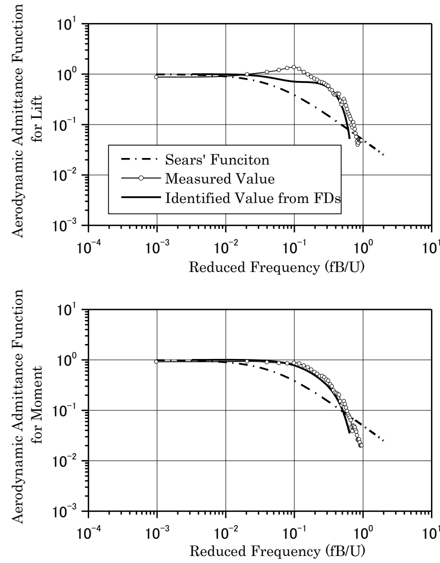


Fig.6 Measured and Identified Aerodynamic Admittance functions of Flat Box Deck Section

The direct measured aerodynamic admittance functions for the flat box deck section are in good agreement with identified ones from FDs. It is obvious that the difference between the obtained ones and Sears' function is due to the different growth of transient aerodynamic force like those in Fig.3

Conclusions

We proposed a new prediction method of aerodynamic admittance function using FDs. The direct measured and identified values for NACA0012 airfoil and flat box deck sections verified its accuracy. Also, aerodynamic admittance function for flat box deck section had a different tendency from Sears' function. It is obvious that the difference between former and latter is caused by the difference in the growth of transient aerodynamic force.

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