



**Dr. Tanaka Seminar**

**No.4**

**Wind Resistant Design  
of  
Long Span Bridge**

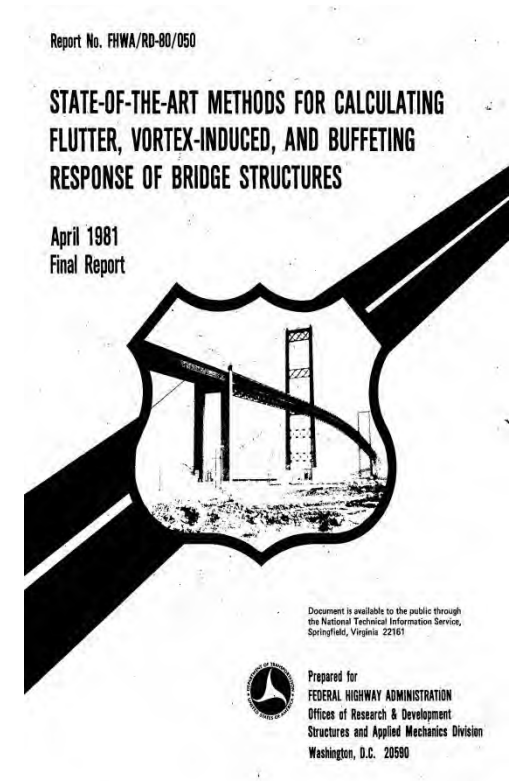
**--- Flutter Analysis ---**

# Method

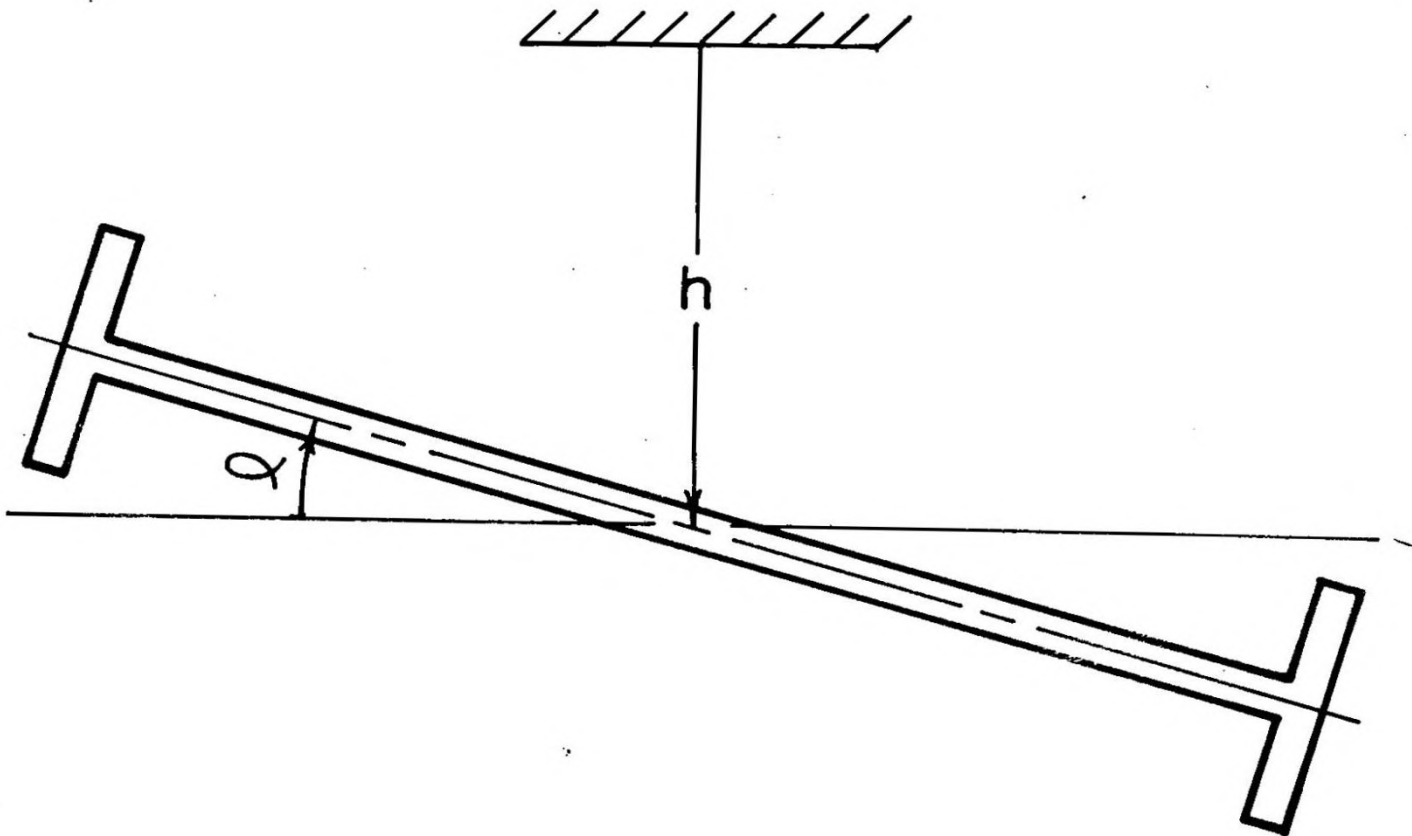
- Flutter analysis is based upon Prof. Scanlan's Method.
  - 1) His method is simple and easy by hands.
  - 2) Preliminary course is assumed.
  - 3) Suspension bridge is main target.

## Reference

R.H. Scanlan : State-of-the-Art Methods for Calculating Flutter, Vortex-Induced, and Buffeting Response of Bridge Structures  
Report No. FHWA/RD-80/050  
April 1981 Final Report



# Analytics of the Flutter Problem



DEGREES OF FREEDOM FOR DECK SECTION

FIGURE 3.1

# The 2-DOF Problem

Consider the bridge deck section as pictured in Fig. 3.1. Let  $h$  represent the vertical deflection of the local c.g. of the section and  $\alpha$  the rotation coordinate (angle) about that c.g.. Let  $m$  represent the mass per unit span and  $I$ , the mass moment of inertia about the c.g., per unit span. Then (neglecting lateral motion as unimportant to flutter) the two sectional equations of motion are

$$(3.1a) \quad M[\ddot{h} + 2 \zeta_h \omega_h \dot{h} + \omega_h^2 h] = L_h$$

$$(3.1b) \quad I[\ddot{\alpha} + 2 \zeta_\alpha \omega_\alpha \dot{\alpha} + \omega_\alpha^2 \alpha] = M_\alpha$$

where  $\zeta_h, \zeta_\alpha$  are the damping ratios-to-critical and  $\omega_h, \omega_\alpha$  are the natural circular frequencies, respectively in  $h$ - and  $\alpha$ - motions, and  $L_h, M_\alpha$  are the aerodynamic force and moment per unit span acting on the section.

its c.g.), the corresponding equations become

$$(3.2a) \quad m[\ddot{h} + a\ddot{\alpha} + 2\zeta_h\omega_h\dot{h} + \omega_h^2 h] = L_h$$

$$(3.2b) \quad I[\ddot{\alpha} + \frac{a}{r_g^2}\ddot{h} + 2\zeta_\alpha\omega_\alpha\dot{\alpha} + \omega_\alpha^2\alpha] = M_\alpha$$

where  $ma$  is the mass unbalance of the section about its c.g., and  $r_g$  is its radius of gyration about the same point.

The aerodynamic force and moment at the c.g. are of the linear, self-excited type and are basically given by

$$(3.3a) \quad \frac{L_h}{m} = H_1 \dot{h} + H_2 \dot{\alpha} + H_3 \alpha$$

$$(3.3b) \quad \frac{M_\alpha}{I} = A_1 \dot{h} + A_2 \dot{\alpha} + A_3 \alpha$$

where the coefficients  $H_i, A_i$  ( $i=1,2,3$ ) are aerodynamic in origin and must be determined experimentally for the particular shape of deck in question.

The coefficients  $H_1$ , pertaining to  $\dot{h}$ , and  $A_2$  and  $A_3$ , pertaining to  $\dot{\alpha}$  and  $\alpha$ , are the direct coefficients, and the others ( $H_2$ ,  $H_3$ , and  $A_1$ ) are the coupling coefficients. In many instances, the direct coefficients prove to be the more important.

$$(3.3c) \quad L_h = \frac{1}{2} \rho U^2 (2B) \left[ K H_1^* \frac{\dot{h}}{U} + K H_2^* \frac{B\dot{\alpha}}{U} + K^2 H_3^* \alpha \right]$$

$$(3.3d) \quad M_\alpha = \frac{1}{2} \rho U^2 (2B^2) \left[ K A_1^* \frac{\dot{h}}{U} + K A_2^* \frac{B\dot{\alpha}}{U} + K^2 A_3^* \alpha \right]$$

where

$$\rho = \text{air density} \quad \left\{ \begin{array}{l} \rho = 0.002378 \text{ slugs/ft}^3 \\ \rho = 1.228 \text{ kg/m}^3 = 1.228 \times 10^{-3} \text{ gm/cm}^3 \end{array} \right.$$

$U$  = cross wind velocity

$$K = \frac{B\omega}{U}$$

$B$  = deck width

$\omega$  = circular frequency of flutter oscillation



and the nondimensional aerodynamic coefficients  $H_i^*$  and  $A_i^*$  bear the following relation to  $H_i$  and  $A_i$ :

$$(3.4) \quad \left\{ \begin{array}{l} H_1^* = \frac{m H_1}{\rho B^2 \omega} ; \\ H_2^* = \frac{m H_2}{\rho B^3 \omega} ; \\ H_3^* = \frac{m H_3}{\rho B^3 \omega^2} ; \end{array} \right. \quad \left\{ \begin{array}{l} A_1^* = \frac{I A_1}{\rho B^3 \omega} \\ A_2^* = \frac{I A_2}{\rho B^4 \omega} \\ A_3^* = \frac{I A_3}{\rho B^4 \omega^2} \end{array} \right.$$

Some examples of the values experimentally obtained for  $H_i^*$  and  $A_i^*$  are illustrated in Figs. 3.2 and 3.3 for decks whose sectional form is sketched on the figures. The plots are given as functions of  $\frac{U}{NB} = \frac{2\pi}{K}$ .

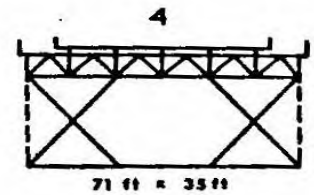
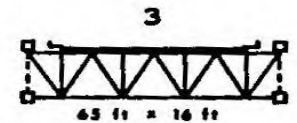
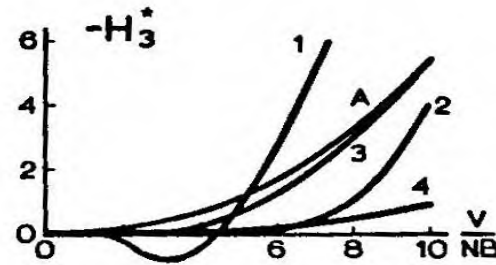
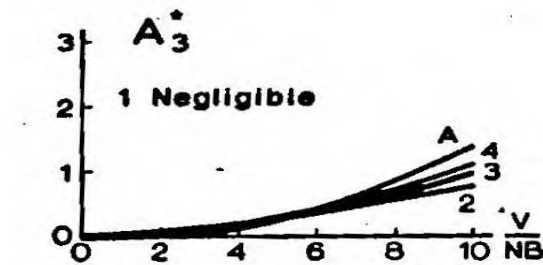
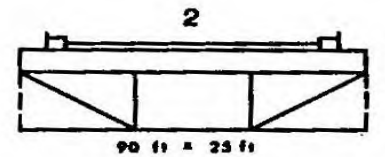
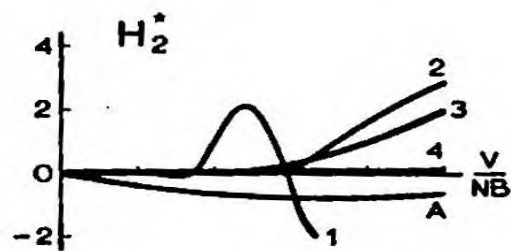
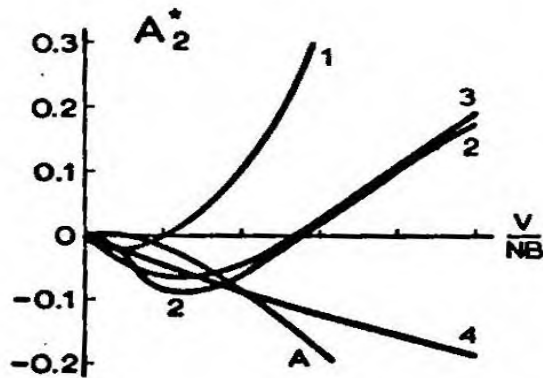
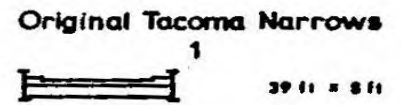
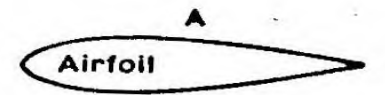
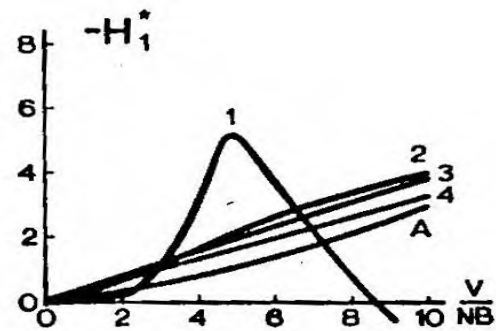
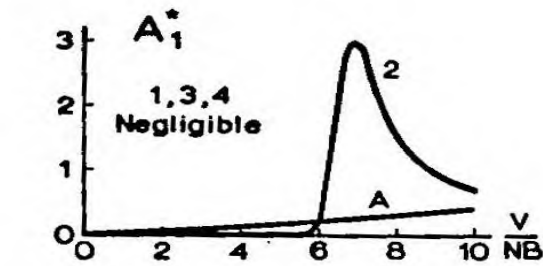


FIGURE 3.2

AERODYNAMIC COEFFICIENTS  
(Flutter Derivatives)

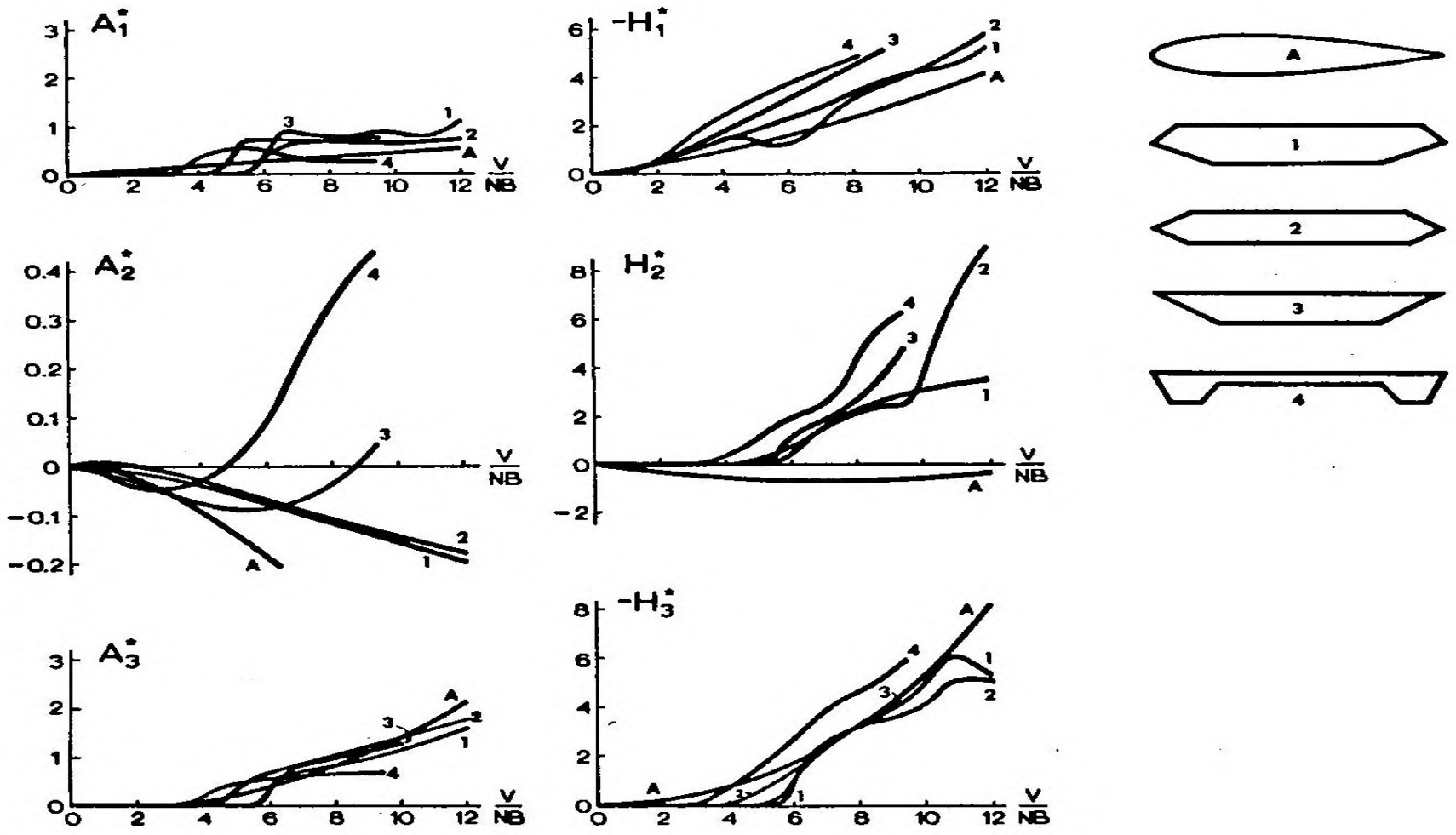


FIGURE 3.3  
AERODYNAMIC COEFFICIENTS

# Flutter Derivatives (1)

It should be noted that the products  $KH_1^*$ ,  $KA_1^*$ , etc., play the roles of flutter derivatives. Consider the term  $H_1 \dot{h}$ , for example. This has the dimension of a force per unit span length. If written in classic aerodynamic lift force form, it would have the appearance

$$\frac{1}{2} \rho U^2 (2B) KH_1^* \frac{\dot{h}}{U} = m H_1 \dot{h} = \frac{1}{2} \rho U^2 (2B) C_L \approx \frac{1}{2} \rho U^2 (2B) \frac{dC_L}{d\bar{\alpha}} \frac{\dot{h}}{U}$$

where  $\bar{\alpha} = \frac{\dot{h}}{U}$  is an "effective" angle of attack.

Thus,

$$K H_1^* = \frac{dC_L}{d\bar{\alpha}}$$

where  $dC_L/d\bar{\alpha}$  is the derivative of a lift coefficient  $C_L$  with respect to angle of attack.

# Flutter Derivatives (2)

It is emphasized again that the values of all the aerodynamic derivatives must be experimentally obtained, and they evolve as functions of reduced velocity  $U/NB$ . Of particular interest is the manner in which the coefficient  $A_2^*$  evolves with  $U/NB$ . This coefficient is proportional to torsional aerodynamic damping, and it plays a central role in many cases of bridge flutter susceptibility, since it often changes sign (from stable to unstable) with increasing  $U/NB$  in certain cases.

# The 3-D Problem

the spanwise position  $x$  and time  $t$ . It will be assumed in this case, that bending and torsion modes of the bridge are independent (uncoupled) from each other. Let  $h(x)$  be the modal deflection form in the lowest bending mode and  $\alpha(x)$  be the modal deflection form in the lowest torsional mode. Then,

$$h(x,t) = h(x) p(t)$$

$$\alpha(x,t) = \alpha(x) q(t)$$

where  $p$  and  $q$  are now the generalized coordinates. The equations of

motion (3.1) for the balanced deck then convert immediately to

$$(3.5a) \quad M_1 [\ddot{p} + 2 \zeta_h \omega_h \dot{p} + \omega_h^2 p] = \frac{1}{2} \rho U^2 (2B) [KC_{11} H_1^* \frac{\dot{p}}{U} + KC_{12} H_2^* \frac{B\dot{q}}{U} + K^2 C_{12} H_3^* q]$$

$$(3.5b) \quad I_1 [\ddot{q} + 2 \zeta_\alpha \omega_\alpha \dot{q} + \omega_\alpha^2 q] = \frac{1}{2} \rho U^2 (2B^2) [KC_{12} A_1^* \frac{\dot{p}}{U} + KC_{22} A_2 \frac{B\dot{q}}{U} + K^2 C_{22} A_3^* q]$$

where

$$(3.6a) \quad M_1 = \int_{\text{span}} m(x) h^2(x) dx$$

$$(3.6b) \quad I_1 = \int_{\text{span}} I(x) \alpha^2(x) dx$$

$$(3.6c) \quad C_{11} = \int_{\text{span}} h^2(x) dx$$

$$(3.6d) \quad C_{12} = \int_{\text{span}} h(x) \alpha(x) dx$$

$$(3.6e) \quad C_{22} = \int_{\text{span}} \alpha^2(x) dx$$

are generalized mass, moment of inertia, and modal factors, respectively.

Generalizations of Eqs. (3.2) for the unbalanced deck are direct but will be

omitted here. Note that  $\int_{\text{span}}$  represents an integral over the entire side-

and main-spans of the bridge (i.e. over whatever constitutes the mode in question).



# Solution of the Flutter Equations

## ① *Pure Torsional Flutter*

If the torsional coefficient  $A_2^*$  reverses sign with increasing  $\frac{U}{NB}$  (see, for example, Fig. 3.4), flutter instability is indicated. This is, in fact, a very common case in practice. To find the wind velocity at flutter ("critical flutter velocity"), proceed as follows.

(In effect, this states that the critical flutter value of  $A_2^*$  is equal to the "scaled" value of mechanical damping  $\zeta_\alpha$  in the  $\alpha$ -mode by a factor equal to twice the ratio of the generalized mechanical inertia to the generalized aerodynamic inertia.)

The corresponding value  $(U/NB)_c$ , is the critical one, and the critical wind velocity is

$$(3.2) \quad U_c = NB \left( \frac{U}{NB} \right)_c$$

Note: If  $A_2^*$  never changes sign, the critical flutter velocity may be high, but in any case, it depends upon solution of the two-degree-of-freedom problem (in both  $p$  and  $q$ ). This is treated below.

In Eq. (3.5b), consider only the damping terms (proportional to  $\dot{q}$ ).

Mechanical damping is just balanced out by aerodynamic damping if:

$$(3.19) \quad I_j (2\zeta_\alpha \omega_\alpha) \dot{q} = \rho U^2 B^2 K C_{22} A_2^* \frac{B \dot{q}}{U}$$

Assuming (as is usually the case), that flutter occurs practically at the same frequency as the first mode, let  $\omega \approx \omega_\alpha$ . Then the condition (3.19) yields the critical value of  $A_2^*$  as

$$(3.20) \quad A_2^* \approx \frac{2 I_j \zeta_\alpha}{\rho B^4 C_{22}}$$

## ② *The 2D-O-F Case for Straight Decks*

This case is more rare and likely to produce flutter only if the lowest bending and torsion frequencies of the bridge are "near" each other. Further, it will not be the important case for a design unless the plot of the coefficient  $A_2^*$  does not reverse sign.

When the conditions for two-degree flutter are satisfied (as, occasionally, for very streamlined bridges), the flutter that can occur is said to be of the "classical" airfoil, or "coupled" type. This is a flutter in which interaction of aerodynamic stiffness terms, rather than damping terms, is the principal mechanism.

The solution proceeds by assuming a sinusoidal response jointly in  $p$  and  $q$  and setting the determinant of the two-equation system (3.5) equal to zero. Details are given in Ref. [3.2]. Letting

$$(3.22) \quad X = \omega/\omega_h$$

where  $\omega$  is the circular flutter frequency, the following two simultaneous equations, with  $X$  as unknown, are obtained:

$$(3.23a) \quad a_4 X^4 + a_3 X^3 + a_2 X^2 + a_1 X + a_0 = 0$$

$$(3.23b) \quad b_3 X^3 + b_2 X^2 + b_1 X + b_0 = 0$$

where the coefficients  $a_i$  and  $b_i$  are the following constants or functions of  $K$ :

$$(3.24a) \quad a_0 = \left(\frac{\omega}{\omega_h}\right)^2$$

$$(3.24b) \quad a_1 = 0$$

$$(3.24c) \quad a_2 = \frac{\omega_\alpha^2}{\omega_h^2} - 4\zeta_h \zeta_\alpha \frac{\omega_\alpha}{\omega_h} - 1 - \frac{\rho B^4}{I_1} C_{22} A_3^*$$

$$(3.24d) \quad a_3 = 2\zeta_\alpha \frac{\omega_\alpha}{\omega_h} \frac{\rho B^2}{M_1} C_{11} H_1^* + 2\zeta_h \frac{\rho B^4}{I_1} C_{22} A_2^*$$

$$(3.24e) \quad a_4 = 1 + \frac{\rho B^4}{I_1} C_{22} A_3^* + \frac{\rho B^2}{M_1} \frac{\rho B^4}{M_1} (C_{12}^2 A_1^* H_2^* - C_{11} C_{22} A_2^* H_1^*)$$

$$(3.25a) \quad b_0 = 2 \zeta_h \left( \frac{\omega_\alpha}{\omega_h} \right)^2 + 2 \zeta_\alpha \frac{\omega_\alpha}{\omega_h}$$

$$(3.25b) \quad b_1 = - \frac{\rho B^2}{M_1} C_{11} H_1^* \frac{\omega_\alpha^2}{\omega_h^2} - \frac{\rho B^4}{I_1} C_{22} A_2^*$$

$$(3.25c) \quad b_2 = - 2 \zeta_\alpha \frac{\omega_\alpha}{\omega_h} - 2 \zeta_h - 2 \zeta_h \frac{\rho B^4}{I_1} C_{22} A_3^*$$

$$(3.25d) \quad b_3 = \frac{\rho B^4}{I_1} C_{22} A_2^* + \frac{\rho B^2}{M_1} C_{11} H_1^* \frac{\rho B^2}{M_1} \frac{\rho B^4}{I_1} (C_{11} C_{22} H_1^* A_3^* - C_{12}^2 A_1^* H_3^*)$$

+



The solution method is as follows. A value of  $K$  is chosen and all the coefficients  $H_i^*$  and  $A_i^*$  are evaluated for that  $K$ . Then  $a_i$  and  $b_i$  in (3.12) and (3.13) are evaluated. These constants are then used in Eqs. (3.11). Eqs. (3.11) are then solved for the values of  $X$  corresponding to the  $K$  chosen. The above process is repeated for a series of values of  $K$ .

Plots of the solutions  $X$  of (3.23a) and (3.23b) are made vs. the  $K$  values used. Where the plot of solutions  $X$  for (3.23a) crosses the plot of solutions  $X$  from (3.23b), the critical flutter condition  $X_c$  exists. The corresponding  $K$  value is

# Ref.[3.2] (1)

Because of its interest in applications, a useful variant on the solution outlined above is now presented in somewhat more detail. Let

$$s = \frac{Ut}{B}$$

be a nondimensional time (or distance). Noting that

$$\left( \dot{\phantom{x}} \right) = \frac{d(\phantom{x})}{dt} = \frac{d(\phantom{x})}{ds} \frac{ds}{dt} = (\phantom{x})' \frac{U}{B}$$

Equations 6.5.2 and 6.5.4 can be reduced to

$$\frac{h''}{B} + 2\zeta_h K_h \frac{h'}{B} + K_h^2 \frac{h}{B} = \frac{\rho B^2}{m} \left[ KH_1^* \frac{h'}{B} + KH_2^* \alpha' + K^2 H_3^* \alpha \right] \quad (6.5.7a)$$

$$\alpha'' + 2\zeta_\alpha K_\alpha \alpha' + K_\alpha^2 \alpha = \frac{\rho B^4}{I} \left[ KA_1^* \frac{h'}{B} + KA_2^* \alpha' + K^2 A_3^* \alpha \right] \quad (6.5.7b)$$

where  $K_h = B\omega_h/U$ ,  $K_\alpha = B\omega_\alpha/U$ .

- Remarks: Eq.(6.5.7a) & (6.5.7b) here are Eq.(3.1a)&(3.2b) .

# Ref.[3.2] (2)

Posing now the solution forms

$$\frac{h}{B} = \frac{h_0}{B} e^{i\omega t} = \frac{h_0}{B} e^{iKs}$$

$$\alpha = \tilde{\alpha}_0 e^{i(\omega t + \phi)} = \alpha_0 e^{i\omega t} = \alpha_0 e^{iKs}$$

Equations 6.5.7 take the form

$$\left[ -K^2 + 2i\zeta_h K_h K + K_h^2 - \frac{\rho B^2}{m} iK^2 H_1^* \right] \frac{h_0}{B} - \left[ \frac{\rho B^2}{m} iK^2 H_2^* + \frac{\rho B^2}{m} K^2 H_3^* \right] \alpha_0 = 0 \quad (6.5.8a)$$

$$\left[ -\frac{\rho B^4}{I} iK^2 A_1^* \right] \frac{h_0}{B} + \left[ -K^2 + 2i\zeta_\alpha K K_\alpha + K_\alpha^2 - \frac{\rho B^4}{I} iK^2 A_2^* - \frac{\rho B^4}{I} K^2 A_3^* \right] \alpha_0 = 0 \quad (6.5.8b)$$

# Ref.[3.2] (3)

Defining an unknown  $X$  as

$$X = \frac{\omega}{\omega_h}$$

and setting the determinant of Eqs. 6.5.8 equal to zero results in a complex polynomial in  $X$  of degree four. This breaks down into the following two equations, assuming that  $X$  is always real at the flutter condition:

From the real part:

$$\begin{aligned} & X^4 \left( 1 + \frac{\rho B^4}{I} A_3^* - \frac{\rho B^2}{m} \frac{\rho B^4}{I} A_2^* H_1^* + \frac{\rho B^2}{m} \frac{\rho B^4}{I} A_1^* H_2^* \right) \\ & + X^3 \left( 2\zeta_\alpha \frac{\omega_\alpha}{\omega_h} \frac{\rho B^2}{m} H_1^* + 2\zeta_h \frac{\rho B^4}{I} A_2^* \right) \\ & + X^2 \left( -\frac{\omega_\alpha^2}{\omega_h^2} - 4\zeta_h \zeta_\alpha \frac{\omega_\alpha}{\omega_h} - 1 - \frac{\rho B^4}{I} A_3^* \right) \\ & + X \cdot 0 \\ & + \left( \frac{\omega_\alpha}{\omega_h} \right)^2 = 0 \end{aligned} \tag{6.5.9a}$$

# Ref.[3.2] (4)

From the imaginary part:

$$\begin{aligned}
 & X^3 \left( \frac{\rho B^4}{I} A_2^* + \frac{\rho B^2}{m} H_1^* + \frac{\rho B^2}{m} \frac{\rho B^4}{I} H_1^* A_3^* - \frac{\rho B^2}{m} \frac{\rho B^4}{I} A_1^* H_3^* \right) \\
 & + X^2 \left( -2\zeta_\alpha \frac{\omega_\alpha}{\omega_h} - 2\zeta_h - 2\zeta_h \frac{\rho B^4}{I} A_3^* \right) \\
 & + X \left( -\frac{\rho B^2}{m} H_1^* \frac{\omega_\alpha^2}{\omega_h^2} - \frac{\rho B^4}{I} A_2^* \right) \\
 & + \left( 2\zeta_h \frac{\omega_\alpha^2}{\omega_h^2} + 2\zeta_\alpha \frac{\omega_\alpha}{\omega_h} \right) = 0
 \end{aligned} \tag{6.5.9b}$$

These two real equations are solved successively for different assumed values of  $K$  and their roots  $X$  are plotted vs.  $K$ . At the point  $(X_c, K_c)$  where the two plots cross, the flutter condition is identified [6-66, 6-67].

The flutter problem as treated above is seen to be a semi-inverse problem since the aerodynamic coefficients are functions of the solution frequency, and a range of frequency parameters  $K$  must therefore be used to survey the solution region.

$$K_c = \frac{B \omega_h X_c}{U_c}$$

i.e., the critical flutter velocity is

$$(3.26) \quad U_c = \frac{B \omega_h X_c}{K_c}$$

# Example of Pure Torsional Flutter

Consider the case where coefficient  $A_2^*$  is the only important torsional aerodynamic coefficient, as given, for example, by Fig. 3.2, Bridge 2. Let the bridge deck mass moment of inertia per unit span be

$$I = 857,000 \text{ lb. sec}^2 (3.9 \times 10^5 \text{ kg sec}^2)$$

with  $B = 100 \text{ ft}$  ( 30.5 meters). If the deck is uniform,  $I_1 = C_{22} I$  and, according to eq. (3.20), the critical value of  $A_2^*$  is given by

$$(A_2^*)_{\text{crit}} = \frac{2 I \zeta_\alpha}{\rho B^4}$$

For a value of mechanical damping of  $\zeta = 0.01$  and air at sea level density

$$(A_2^*)_{\text{crit}} = \frac{2 \cdot 857000 \cdot (0.01)}{(0.002378) \cdot 100^4} = 0.072 \quad (\text{nondimensional})$$

According to Fig. 3.2 this corresponds to a critical  $U/NB$  value of 7.1 which, for a natural torsional frequency of  $N = 0.2$  Hz, corresponds to a flutter velocity of  $U_c = 142$  ft/sec = 96.8 mph (= 156 km/hr).



## Example of Two-Degree Flutter

The mechanical damping  $\zeta_r$  and  $\zeta_h$  will both be taken as 0.01; the aerodynamic coefficients will be taken from Fig. 3.3 and are as listed below (for bridge section #2). The deck width  $B=100$  ft (30.5 m), while the span  $L=4,000$  ft. (1220 m). The vertical and torsional modes will be assumed to be half sine waves, therefore  $C_{11} = C_{12} = C_{22} = L/2 = 2,000$  ft. (610 m). The constant sectional moment of inertia per unit span will be taken as  $I = 857,000$  lb. sec<sup>2</sup> ( $3.9 \times 10^5$  kg sec<sup>2</sup>) while the mass of the deck per unit span will be  $M = 711.8$  lb sec<sup>2</sup>/ft<sup>2</sup> ( $3481$  kg sec<sup>2</sup>/m<sup>2</sup>). These lead to:

$$\left(\frac{\rho B^4}{I_1}\right) c_{11} = \left(\frac{\rho B^4}{I_1}\right) c_{12} = \left(\frac{\rho B^4}{I_1}\right) c_{22} = 0.277$$

$$\left(\frac{\rho B^2}{M_1}\right) c_{11} = \left(\frac{\rho B^2}{M_1}\right) c_{12} = \left(\frac{\rho B^2}{M_1}\right) c_{22} = 0.0334$$

It will be assumed that  $\omega_\alpha = 2 \omega_h$  and that  $\omega_h = 2\pi(.1 \text{ Hz})$ .

U/NB	$A_1^*$	$A_2^*$	$A_3^*$	$H_1^*$	$H_2^*$	$H_3^*$
2	0	0	0	-0.67	0	0
4	0	-0.03	0	-1.50	0	-0.05
6	0.75	-0.05	0.50	-2.05	0.7	-1.25
8	0.70	-0.10	1.00	-3.25	2.25	-3.35
10	0.68	-0.14	1.46	-4.25	4.25	-4.00
12	0.70	-0.16	1.69	-5.50	8.90	-5.00

CUBIC EQUATION

$$b_0 = 2(.01)(2)^2 + 2(.01)^2 = 0.12$$

$$b_1 = -0.1336 H_1^* - 0.277 A_2^*$$

$$b_2 = -0.06 - 0.00554 A_3^*$$

$$b_3 = 0.0277 A_2^* + 0.0334 H_1^* + .00925 [H_1^* A_3^* - A_1^* H_3^*]$$

U/NB = 12 (K = .523)

$$b_0 = .12$$

$$b_1 = (-.1336)(-5.50) - 0.277(-.16) = .7791$$

$$b_2 = -.06 - 0.00554(1.69) = -.06936$$

$$b_3 = (.277)(-.16) + [.0334)(-.5.5) \\ + .00925 [(-5.5)(1.69) + 5(.70)] \\ = -.2816$$

$$(-.2816)x^3 - (.06936)x^2 + (.7791)x + .12 = 0$$

$$x^3 + (.2462)x^2 - 2.7667x - .4261 = 0$$

$$x = 1.62$$

(Negative roots neglected)

$$\underline{U/NB = 10} \quad (K = .628)$$

$$b_0 = .12$$

$$b_1 = (-.1336)(-4.25) - (.277)(-.14) = 0.6066$$

$$b_2 = -.06 - .00554(1.46) = -.06809$$

$$\begin{aligned} b_3 &= (.277)(-.14) + .0334(-4.25) \\ &\quad + .00925 [(-4.25)(1.46) + 4(.68)] \\ &= -.2130 \end{aligned}$$

$$(-.2130)x^3 - (.06809)x^2 + (.6066)x + .12 = 0$$

$$x^3 + .3197 x^2 - 2.848 x - .5634 = 0$$

$$x = 1.63$$

$$\underline{U/NB} = 8 \quad (K = .785)$$

$$b_0 = .12$$

$$b_1 = (-.1336)(-3.25) - .277(-.10) = 0.4619$$

$$b_2 = -.06 - (.00554)(1.00) = -.06554$$

$$\begin{aligned} b_3 &= (.277)(-.10) + (.0334)(-3.25) \\ &\quad + .00925 [(-3.25) + 3.35(.70)] \\ &= -.1446 \end{aligned}$$

$$(-.1446)x^3 - (.06554)x^2 + .4619x + .12 = 0$$

$$x^3 + (.4533)x^2 - (3.194)x - .8299 = 0$$

$$x = 1.71$$

$$\underline{U/NB} = 6 \quad (K = 1.05)$$

$$b_0 = .12$$

$$b_1 = (-.1336)(-2.05) - (.277)(-.05) = .2877$$

$$b_2 = -.06 - (.00554)(0.50) = -.06277$$

$$\begin{aligned} b_3 &= (.277)(-.05) + (.0334)(-2.05) \\ &\quad + .00925 [(-2.05)(.50) - (.75)(-1.25)] \\ &= -0.0831 \end{aligned}$$

$$(-.0831)x^3 - (.06277)x^2 + (.2877)x + .12 = 0$$

$$x^3 + (.7554)x^2 - 3.462x - 1.444 = 0$$

$$x = 1.73$$

$$\underline{U/NB} = 4 \quad (K = 1.57)$$

$$b_0 = .12$$

$$b_1 = (-.1336)(-1.50) - .277(-.03) = .2087$$

$$b_2 = -.06 - .0054(0) = -.06$$

$$b_3 = -.00831 + .0334(-1.5) = -.0584$$

$$(-.0584)x^3 - (.06)x^2 + (.2087)x + .12 = 0$$

$$x^3 + (1.027)x^2 - (3.574)x - 2.055 = 0$$

$$x = 1.73$$

$$\underline{U/NB} = 2 \quad (K = 3.14)$$

$$b_0 = .12$$

$$b_1 = (-.1336)(-0.67) - (.277)(0) = .08951$$

$$b_2 = -.06$$

$$b_3 = (.0334)(-.67) = -.02238$$

$$(-.02238)x^3 - (.06)x^2 + (.08951)x + .12 = 0$$

$$x^3 + 2.681x^2 - 4.00x - 5.362 = 0$$

$$x = 1.63$$



Quartic Equation

$$a_0 = (2)^2 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - .277 A_3^*$$

$$a_3 = .00134 H_1^* + .00554 A_2^*$$

$$a_4 = 1 + .277 A_3^* + .00953 [A_1^* H_2^* - A_2^* H_1^*]$$

U/NB = 12 (K = .523)

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - (2.77)(1.69) = -5.469$$

$$a_3 = (.00134)(-5.50) + (.00554)(-.16) = -.00826 \text{ (neglig.)}$$

$$a_4 = 1 + (.277)(1.69) + .00953 [(.70)(8.9) - 5.50(.16)]$$
$$= 1.519$$

$$(1.519)x^4 - 5.469x^2 + 4 = 0$$

$$x^2 = \frac{5.469 \pm \sqrt{(5.469)^2 - 4(4)(1.519)}}{2(1.519)}$$

$$x^2 = 2.579, \quad 1.021$$

$$x = 1.61, \quad 1.01$$

$$\underline{U/NB} = 10 \quad (K = .628)$$

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - (.277)(1.46) = -5.405$$

$$a_3 = \text{negligible}$$

$$a_4 = 1 + (.277)(1.46) + .00953 [(.68)(4.25) - (4.25)(.14)] \\ = 1.426$$

$$1.426 x^4 - 5.405 x^2 + 4 = 0$$

$$x^2 = \frac{5.405 \pm \sqrt{(5.405)^2 - 4(4)(1.426)}}{2(1.426)}$$

$$x^2 = 2.782, 1.008$$

$$x = 1.67, 1.00$$

$$\underline{U/NB} = 8 \quad (K = .785)$$

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - (.277)(1.00) = -5.278$$

$$a_3 = \text{negligible}$$

$$\begin{aligned} a_4 &= 1 + (.277)(1.00) + .00953 [(.70)(2.25) - (.10)(3.25)] \\ &= 1.289 \end{aligned}$$

$$1.289 x^4 - 5.278 x^2 + 4 = 0$$

$$x^2 = \frac{5.278 \pm \sqrt{(5.278)^2 - 4(4)(1.289)}}{2(1.289)}$$

$$x^2 = 3.091 \quad , \quad 1.004$$

$$x = 1.76 \quad , \quad 1.00$$

$$\underline{U/NB} = 6 \quad (K = 1.05)$$

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - .277(.50) = -5.139$$

$$a_3 = \text{negligible}$$

$$a_4 = 1. + (.277)(.50) + .00953 [(.75)(.70) - 2.05 (.05)]$$
$$= 1.143$$

$$(1.143) x^4 - 5.139 x^2 + 4 = 0$$

$$x^2 = \frac{5.139 \pm \sqrt{(5.139)^2 - 4(4)(1.43)}}{2(1.143)}$$

$$x^2 = 3.495 \quad , \quad 1.001$$

$$x = 1.87 \quad , \quad 1.00$$

$$\underline{U/NB} = 4 \quad (K = 1.57)$$

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008$$

$$a_3 = \text{negligible}$$

$$a_4 = 1.000$$

$$x^2 = \frac{5.0008 \pm \sqrt{(5.0008)^2 - 16}}{2}$$

$$x^2 = 4.001 \quad , \quad .99973$$

$$x = 2.000 \quad , \quad 1.000$$

$$\underline{U/NB} = 2.0 \quad (K = 3.14)$$

$$a_0 = 4$$

$$a_1 = 0$$

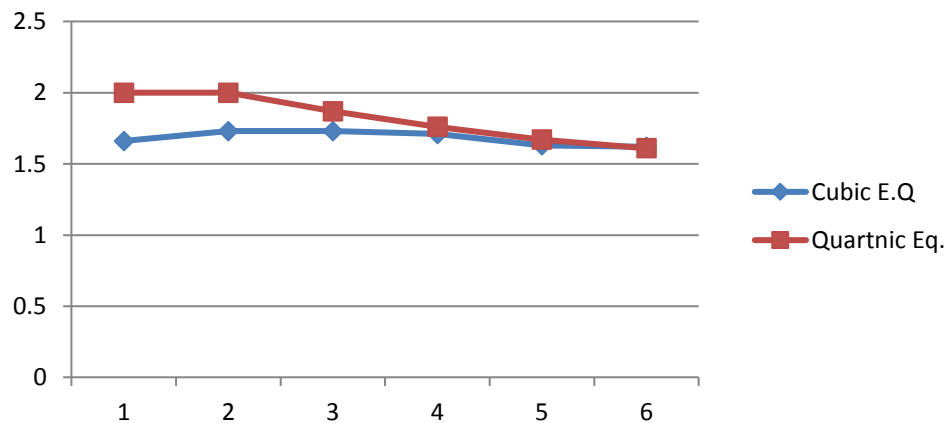
$$a_2 = -5.0008$$

$$a_4 = 1.00$$

$$X = 2.000, 1.000$$

$$\text{VALUES OF } X = \frac{\omega}{\omega_h}$$

<u>U/NB</u>	<u>K</u>	<u>Cubic Eq.</u>	<u>Quartic Eq.</u>	
2	3.14	1.66	1.00	2.00
4	1.57	1.73	1.00	2.00
6	1.05	1.73	1.00	1.87
8	.785	1.71	1.00	1.76
10	.628	1.63	1.00	1.67
12	.523	1.62	1.01	1.61





Cubic equation intersects quartic equation at

$$X = 1.62 \quad K = .53 \quad (U/NB = 11.7)$$

Therefore,

$$U_c = \frac{B\omega h^X}{K_c}$$

$$U_c = \frac{(100 \text{ ft}) [2\pi (.1)] 1.62}{0.53}$$

$$U_c = 190.3 \text{ ft/sec} = 129.7 \text{ mph} \\ (58.0 \text{ m/sec})$$

# Conclusions

- Scanlan's method is easy and accurate.
- Flutter derivatives are main roles for flutter analysis.
  - Therefore to obtain flutter derivatives by wind tunnel tests are very important.
  - Bridge designers should practice flutter analysis by hands at least one time.

# References

- [3.1] Scanlan, R.H.: "Recent Methods in the Application of Test Results to the Wind Design of Long, Suspended-Span Bridges," Report No. FHWA-RD-75-115, Federal Highway Administration Office of Research and Development, Washington, DC, 1975.
- [3.2] Simiu, E. and Scanlan, R.H.: Wind Effects on Structures, Wiley, New York, 1978.
- [3.3] Scanlan, R.H. and Tomko, J.J.: "Airfoil and Bridge Deck Flutter Derivatives," Jnl. Eng. Mech. Div. ASCE, Vol. 97, No. EM6, Dec. 1971, pp. 1717-1737.

## PART—2

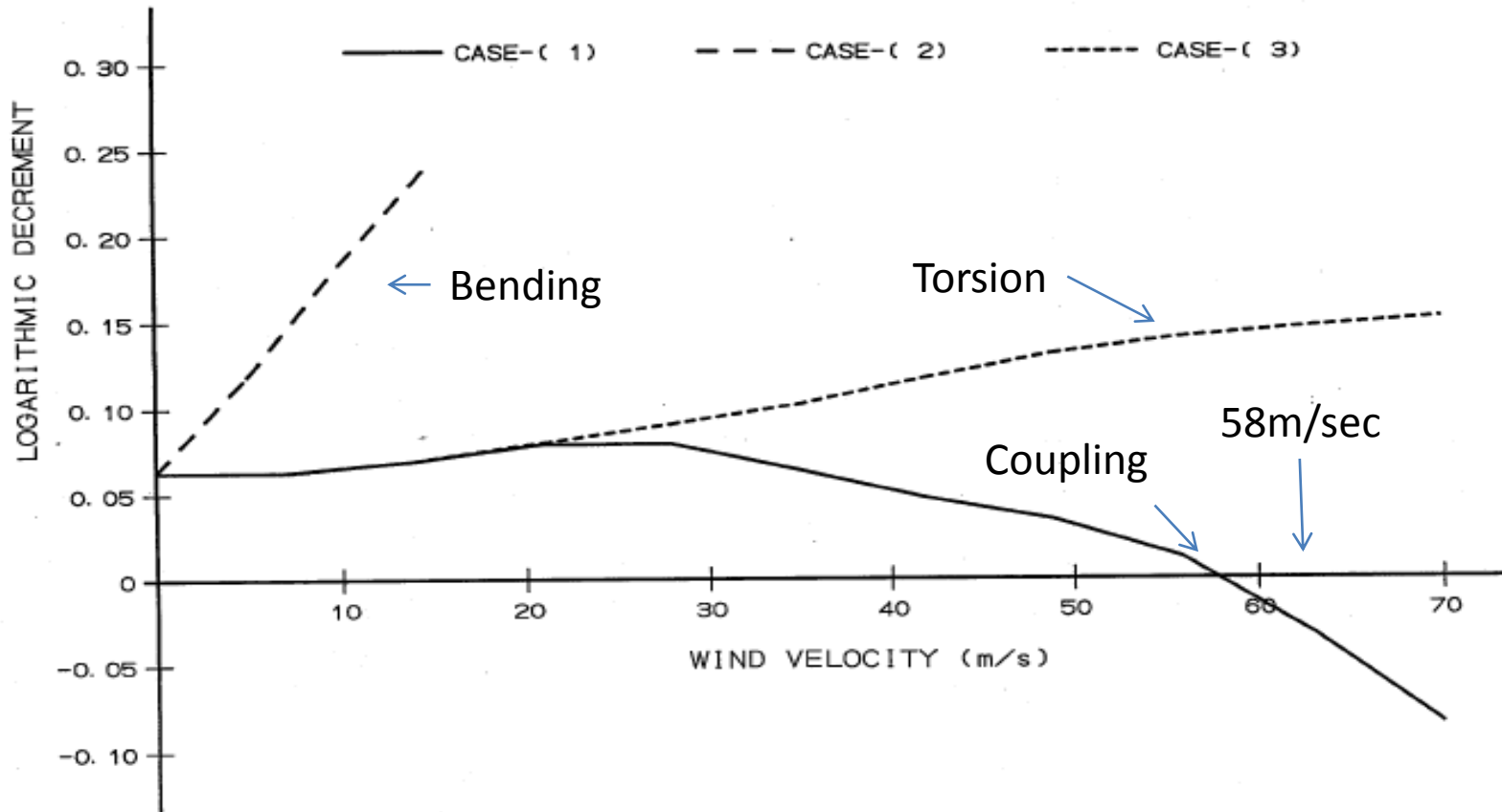
- Flutter analysis method will be applied to same problem.
  - The method was developed by Hiroshi TANAKA at Hitachi Zosen in 1990.
  - He received 'Tanaka Prize' by the following paper.

Paper

Multi-Mode Flutter Analysis and  
Two & Three Dimensional Model Tests on  
Bridges with Non-analogous Modal Shapes  
JSCE J.Struct.Mech.Earthquake Eng. July 1993

# Results of Dr. Tanaka

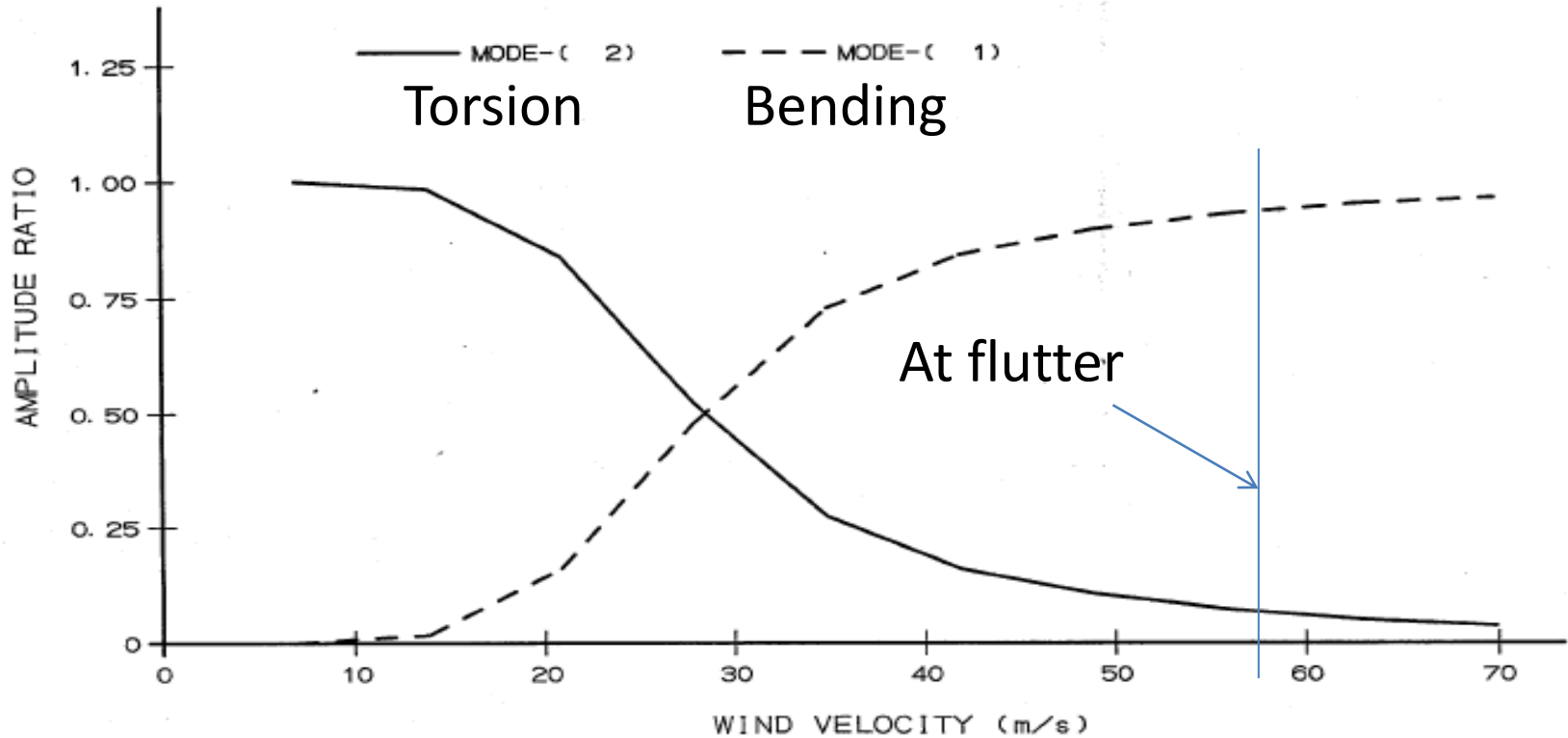
•• SCANLAN'S EXAMPLE (FHWA/RD-80/050) P. 39-47; JAN. -1990 ••



MULTI-DISPLAY FOR ALL CASES: ( 1)-----( 3)

# Amplitude Ratio of mode-1 and 2

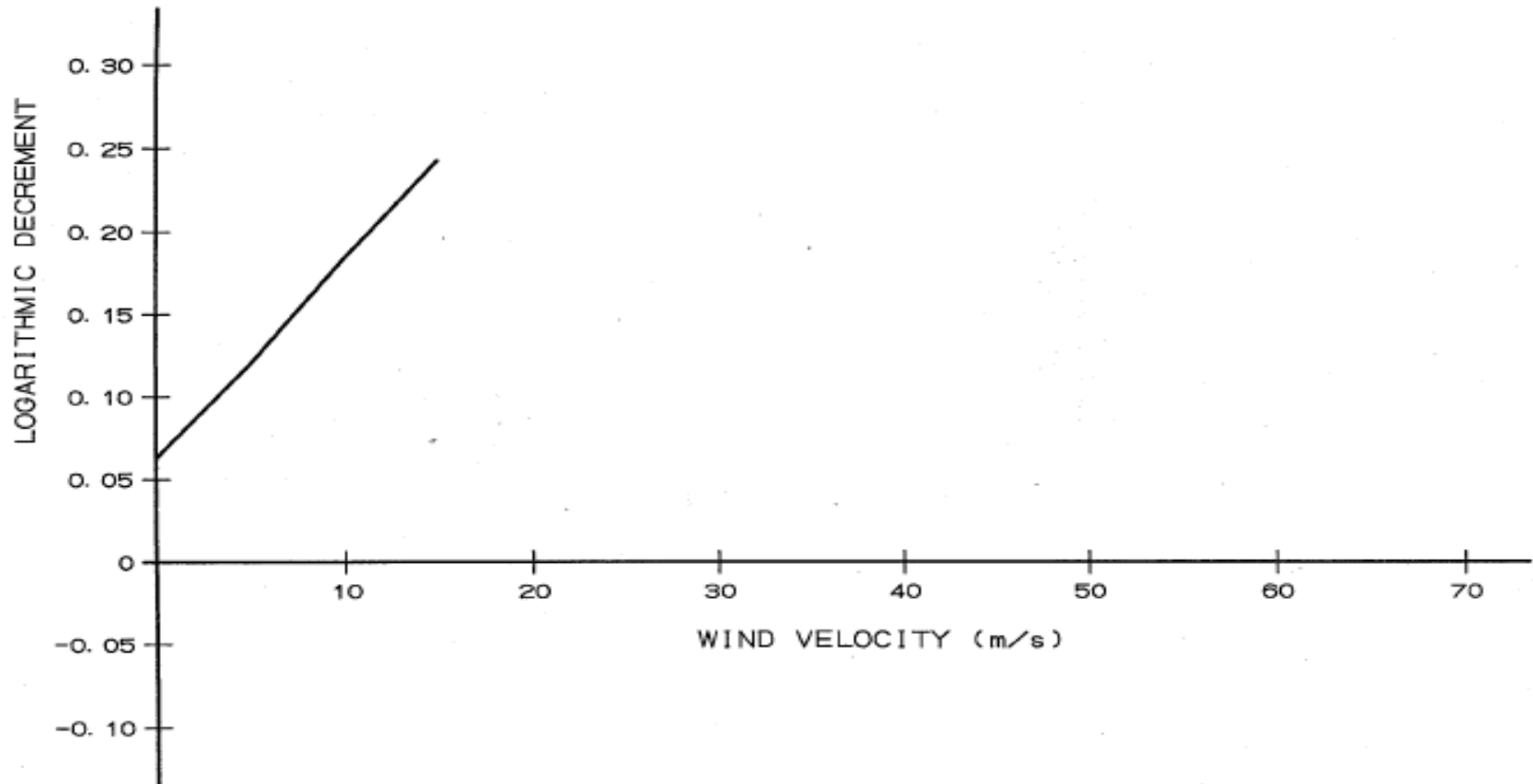
•• SCANLAN'S EXAMPLE (FHWA/RD-80/050) P. 39-47; JAN. -1990 ••



CASE-( 1); COUPLING FLUTTER; MODES-(1, 2)

# Change of Damping (Bending)

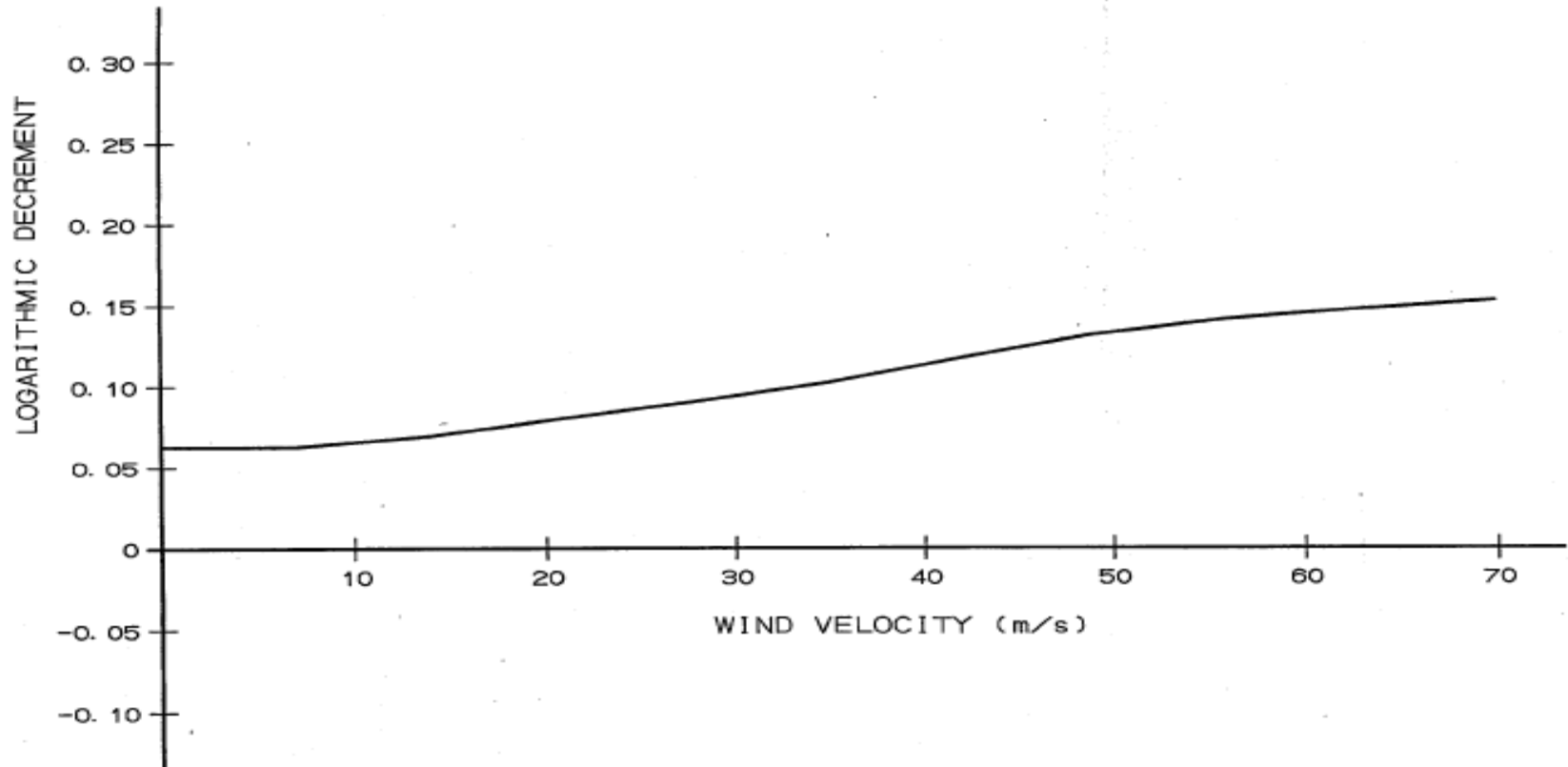
•• SCANLAN'S EXAMPLE (FHWA/RD-80/050) P. 39-47; JAN. -1990 ••



CASE-( 2); VERTICAL FLUTTER; MODE-1 ONLY

# Change of Damping (Torsion)

•• SCANLAN'S EXAMPLE (FHWA/RD-80/050) P. 39-47; JAN. -1990 ••

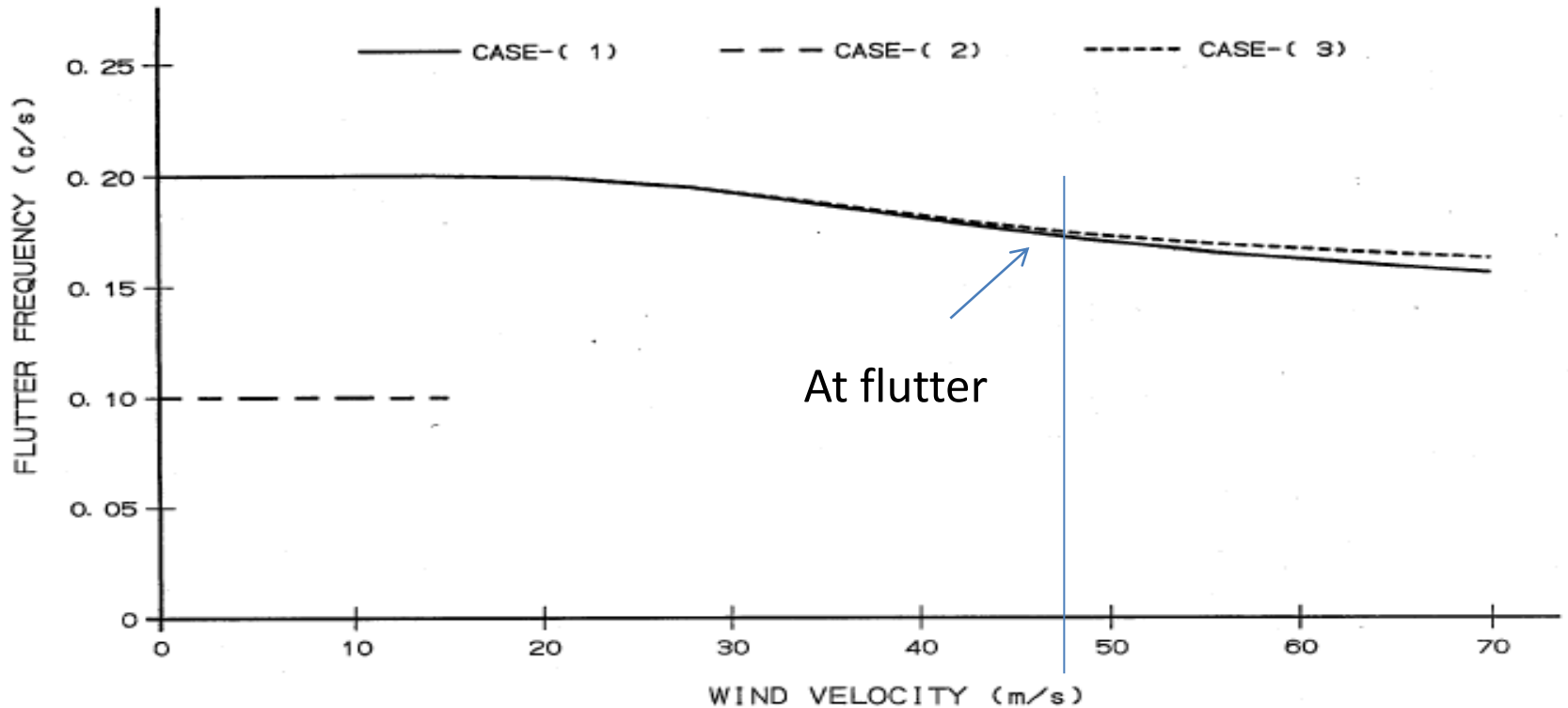


CASE-( 3 ); TORSIONAL FLUTTER: MODE-2 ONLY



# Flutter Frequency

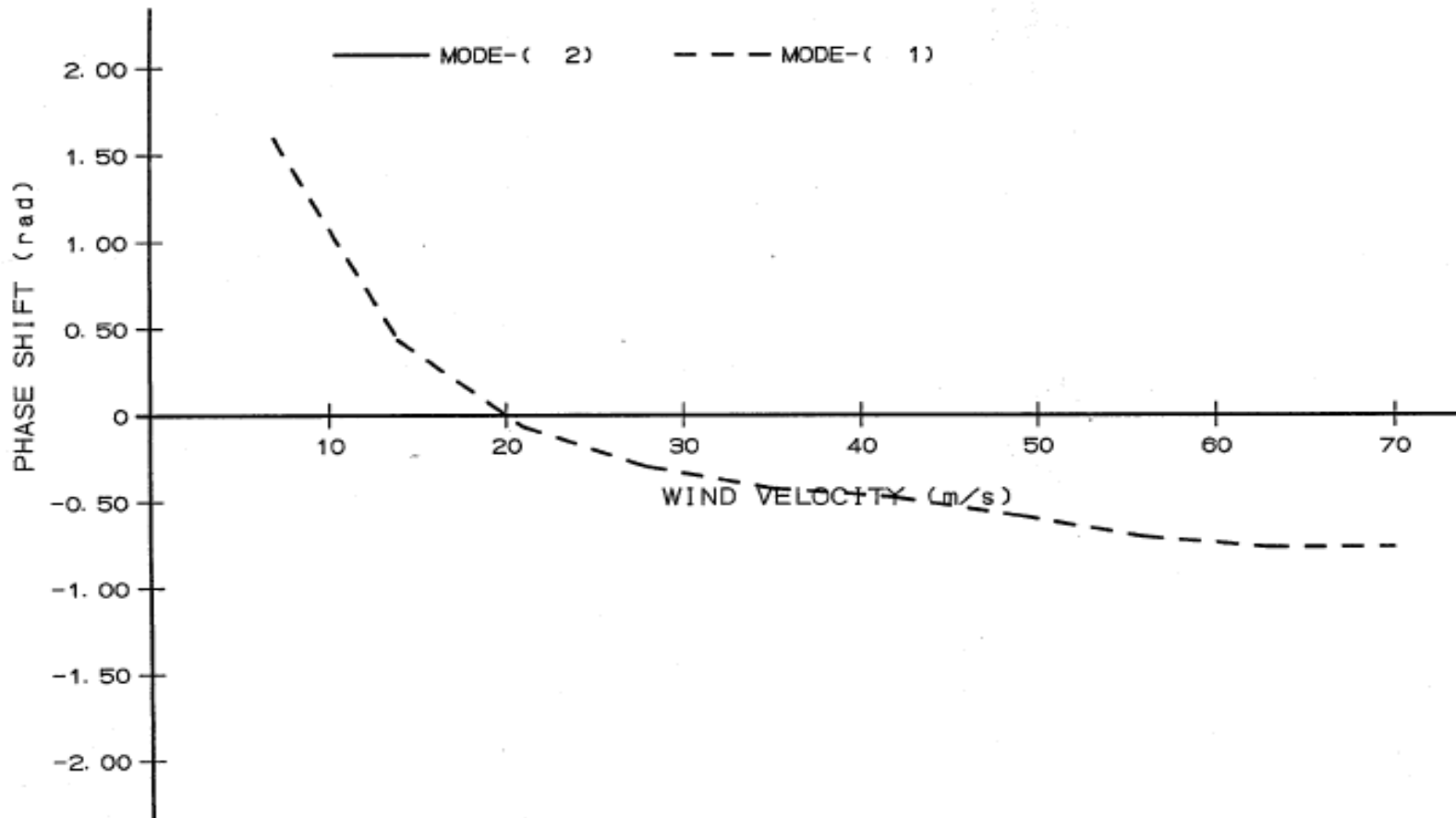
•• SCANLAN'S EXAMPLE (FHWA/RD-80/050) P. 39-47; JAN. -1990 ••



MULTI-DISPLAY FOR ALL CASES: ( 1)-----( 3)

# Phase Shift (rad)

•• SCANLAN'S EXAMPLE (FHWA/RD-80/050) P. 39-47; JAN. -1990 ••



CASE-( 1 ): COUPLING FLUTTER: MODES-(1, 2)

# Input Form

PAGE = 1

PAGE = 2

	1	2	3	4	5	6	7	8
1	TITL	** SCANLAN'S EXAMPLE	(FHWA/RD-80/050)	P. 39-47;	JAN. -1990	**		
2	CNTL	98	1	1	1			
3	UNIT							
4	MATL	1	20.5800	79.3800				
5	SECT	1	334.2780	1	1488	1	115609	37455.60
6	NODE	1	0			1	1	1
7	NODE	2	122					
8	NODE	3	244					
9	NODE	4	366					
10	NODE	5	488					
11	NODE	6	610					
12	NODE	7	732					
13	NODE	8	854					
14	NODE	9	976					
15	NODE	10	1098					
16	NODE	11	1220			1	1	1
17	MEMB	1	1	2	1	1		0.0
18	MEMB	2	2	3	1	1		0.0
19	MEMB	3	3	4	1	1		0.0
20	MEMB	4	4	5	1	1		0.0
21	MEMB	5	5	6	1	1		0.0
22	MEMB	6	6	7	1	1		0.0
23	MEMB	7	7	8	1	1		0.0
24	MEMB	8	8	9	1	1		0.0
25	MEMB	9	9	10	1	1		0.0
26	MEMB	10	10	11	1	1		0.0
27	CLAS	.0628	1	11				
28	SELF	1	10	1				
29	HST1	2.0	-1.34	4.0	-3.00	6.0	-4.10	8.0
30	HST1	12.0	-11.00					
31	HST2	2.0	0.00	4.0	0.00	6.0	-1.40	8.0
32	HST2	12.0	-17.80					
33	HST3	2.0	0.00	4.0	0.10	6.0	2.50	8.0
34	HST3	12.0	10.00					
35	AST1	2.0	0.00	4.0	0.00	6.00	-1.50	8.0
36	AST1	12.0	-1.40					
37	AST2	2.0	0.00	4.0	-0.06	6.00	-0.10	8.0
38	AST2	12.0	-0.32					
39	AST3	2.0	0.0	4.0	0.0	6.0	1.00	8.0
40	AST3	12.0	3.38					
41	FLUT	1COUPLING FLUTTER; MODES-(1, 2)						
42	FACT	1.22794	50	9999.				
43	COPL	5	70.0	10	2	1		1
44	LOD2	1	10	1	1		0.00001	30.5
45	FLUT	2VERTICAL FLUTTER; MODE-1 ONLY						
46	FACT	1.22794	50	9999.				
47	COPL	5	15.0	3	1			1
48	LOD2	1	10	1	1		0.00001	30.5
49	FLUT	3TORSIONAL FLUTTER; MODE-2 ONLY						
50	FACT	1.22794	50	9999.				

	1	2	3	4	5	6	7	8
51	COPL	5	70.0	10	2			1
52	LOD2	1	10	1	1		0.00001	30.5
53	END							

# Conclusions

- Tanaka Method will give you many information on flutter.
- Input is small.
- Calculation time is less than 10 sec.
- You can understand flutter phenomenon.

THANK YOU !!