

# **Flutter and Gust Response Analyses of the Messina Strait Bridge - Benchmark Study -**

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# the Messina Straits Bridge



Sicily

Southern  
Mainland Italy

Longest span	3300m (Akashi-Kaikyo 1991m)
Tower height	382.6m (Akashi-Kaikyo 298.3m)
Deck width	60.4m (Akashi-Kaikyo 35.5m)
Cable diameter	4 × 1.24m (Akashi-Kaikyo 2 × 1.12m)
Design wind speed	75m/s (Akashi-Kaikyo 80m/s)

Reference from J. Ramsden, Proc. of Bridge Engineering 2 Conference 2009.

# Deck Section

Deck weight 18.1 t/m

Cable weight 37 t/m



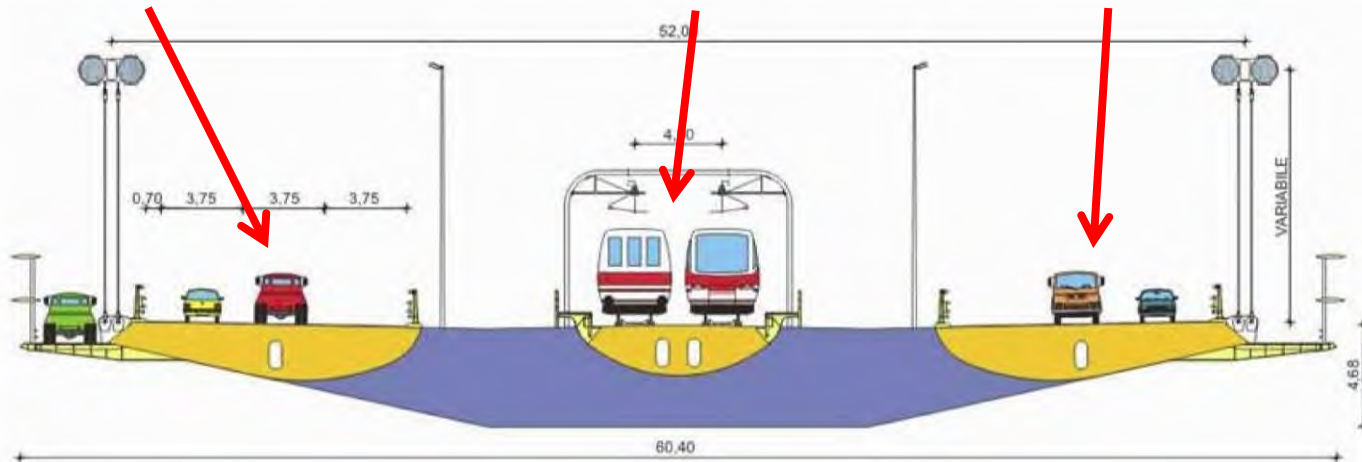
**Total weight  
55.1t/m**

Chord	Material	Total weight	N. Boxes	Road lane	N. Tracks
60 m	Steel	66500 t	3	6 + 2	2

Roadway Box Girder

Railway Box Girder

Roadway Box Girder

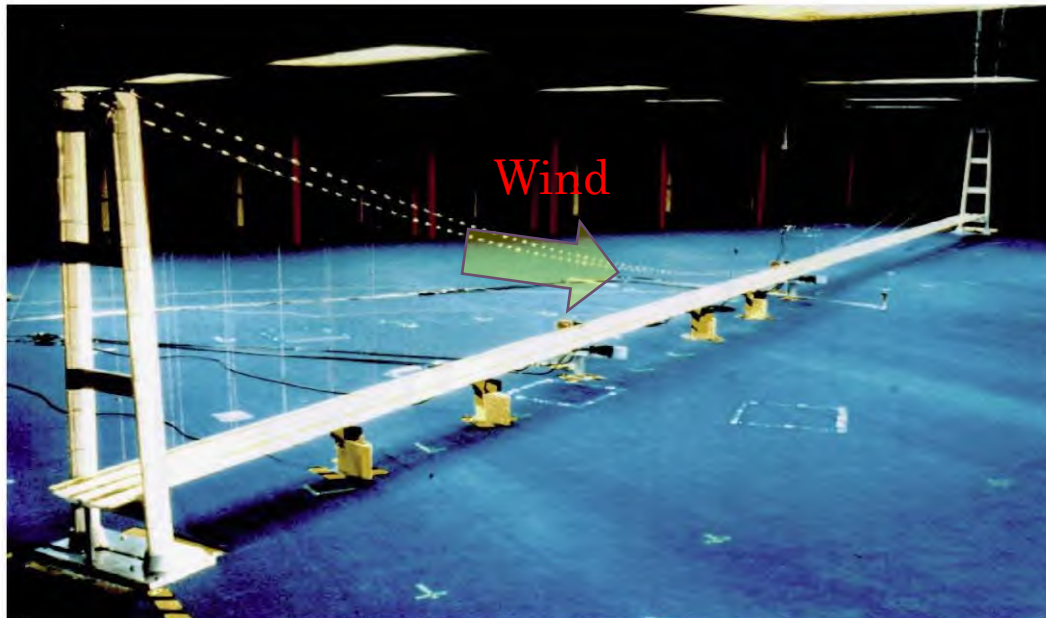


# What is the benchmark study?

- ◆ The aerodynamic study (wind tunnel test and analysis) of the Messina Straits Bridge has been carried out by Prof. Diana's research group of Politecnico di Milano, Italy.
- ◆ The structural and aerodynamic data has been disclosed on the Internet.
- ◆ We can be compared with their flutter & gust response analyses and experimental results.
- ◆ In this analysis, we use the aerodynamic analysis codes developed by Dr.Yamamura and Dr.Tanaka.

# Full Aeroelastic Model Test in Boundary Layer Turbulent Flow

Full aeroelastic model: DMI (1992)



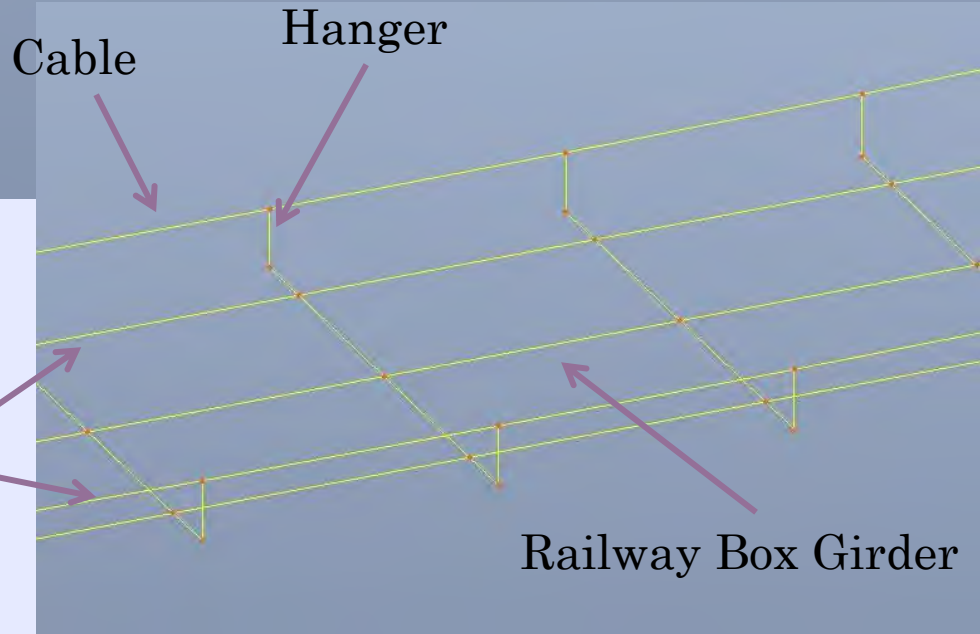
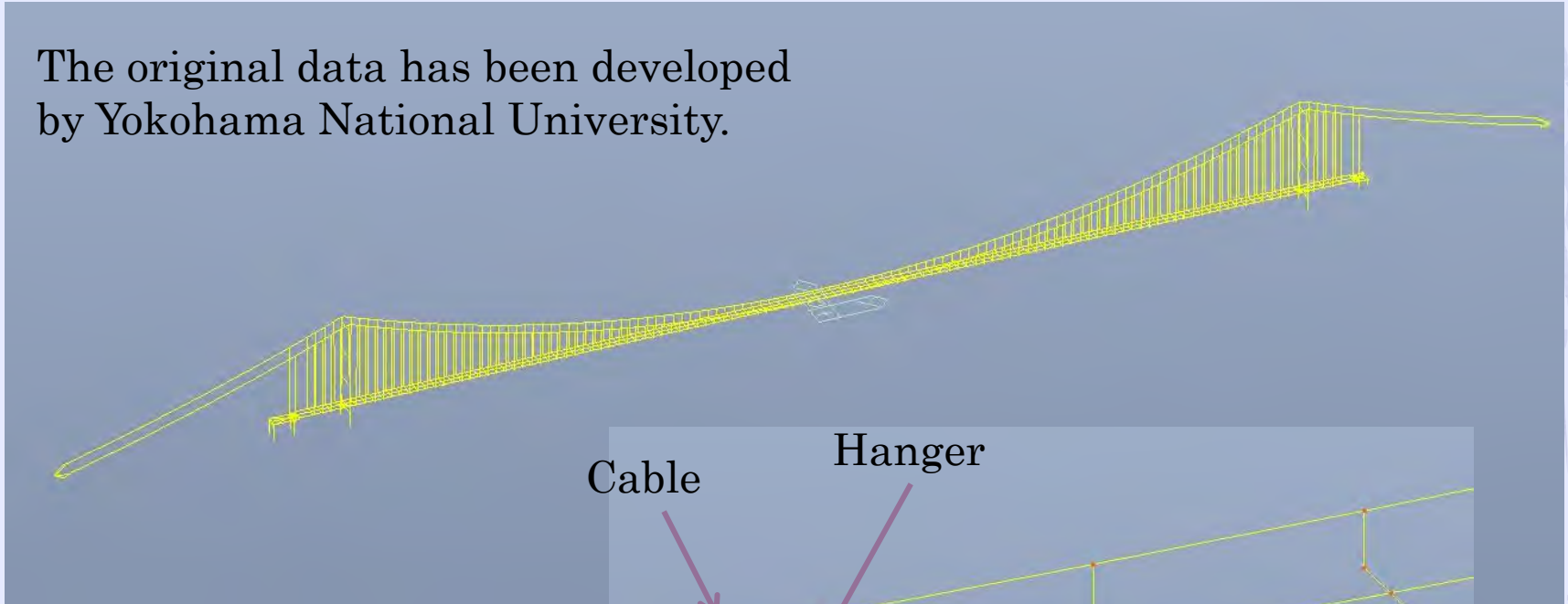
Reference from G. Diana : Messina Bridge Project – Technical Challenges -, 2006

# Outline of Flutter Analysis

- ◆ The flutter analysis is the **3-dimensional (3D) flutter analysis** of multi-degree of freedom system with a 3-dimensional frame model (**Mult-Mode Flutter Analysis**).
- ◆ Self-excited forces are formulated using **Scanlan's conventions** (flutter derivatives:  $P^*_i$ ,  $H^*_i$ ,  $A^*_i$ ).
- ◆ **The benchmark data (experimental data)** of the flutter derivatives for **lift** and **moment forces** are used. The flutter derivatives for **drag force** are calculated by **quasi steady theory**.
- ◆ The flutter analysis is carried out using **modal analysis approach**. The lowest two or three bending modes and the lowest torsional mode are selected as **the key modes of coupling flutter modes**.
- ◆ The structural damping in air flow is calculated by **complex Eigen value analysis**. From the structural damping, **the flutter onset velocity** are identified.

# FEM Model (Beam Element)

The original data has been developed by Yokohama National University.



Roadway Box Girder

Railway Box Girder



# Natural Frequency Analysis

Mode No. of NTI Model	Natural Freq.(Hz)			Equivalent Mass (t/m or t <sup>2</sup> /m)	Mode description
	NTI	YNU AM1 <sup>2)</sup>	Prof. Diana's group <sup>3)</sup>		
<b>1</b>	<b>0.031</b>	<b>0.031</b>	<b>0.033</b>	<b>52.6</b>	<b>Sym. and horizontal</b>
<b>2</b>	<b>0.059</b>	<b>0.059</b>	<b>0.059</b>	<b>33.8</b>	<b>Asym. and horizontal</b>
<b>3</b>	<b>0.063</b>	<b>0.064</b>	<b>0.061</b>	<b>60.2</b>	<b>Asym. and vertical</b>
<b>4</b>	<b>0.078</b>	<b>0.078</b>	<b>0.080</b>	<b>57.0</b>	<b>Sym. and vertical</b>
5	0.084			—	—
<b>6</b>	<b>0.090</b>	<b>0.076</b>	<b>0.081</b>	<b>32,421</b>	<b>Asym. and torsional</b>
7	0.091			—	—
8	0.096			—	—
9	0.098			—	—
<b>10</b>	<b>0.101</b>	<b>0.093</b>	<b>0.097</b>	<b>31,203</b>	<b>Sym. and torsional</b>

# Vibration Mode

- 1<sup>st</sup> mode of sway motion -



Side view

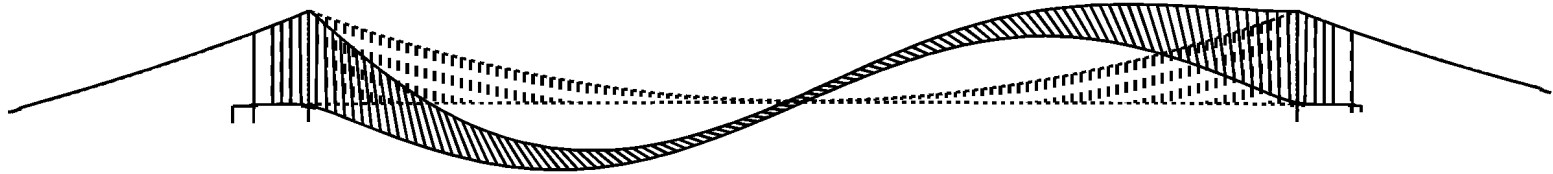


View from upper side

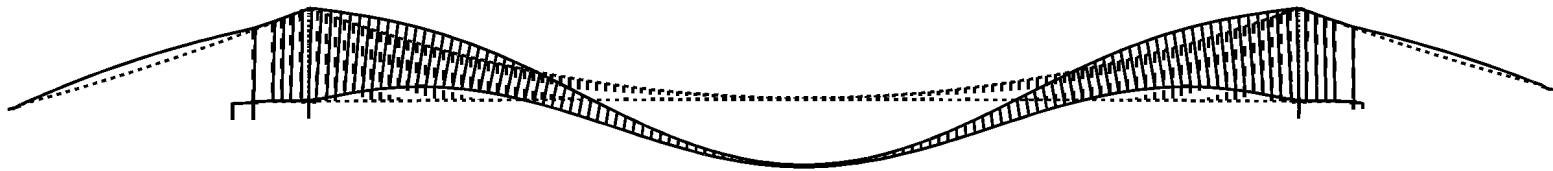
$$f = 0.031\text{Hz}, T = 32.2\text{sec}, M_{\text{eq}} = 52.6\text{t/m}$$

# Vibration Mode

- 1<sup>st</sup> & 2<sup>nd</sup> modes of bending motion -



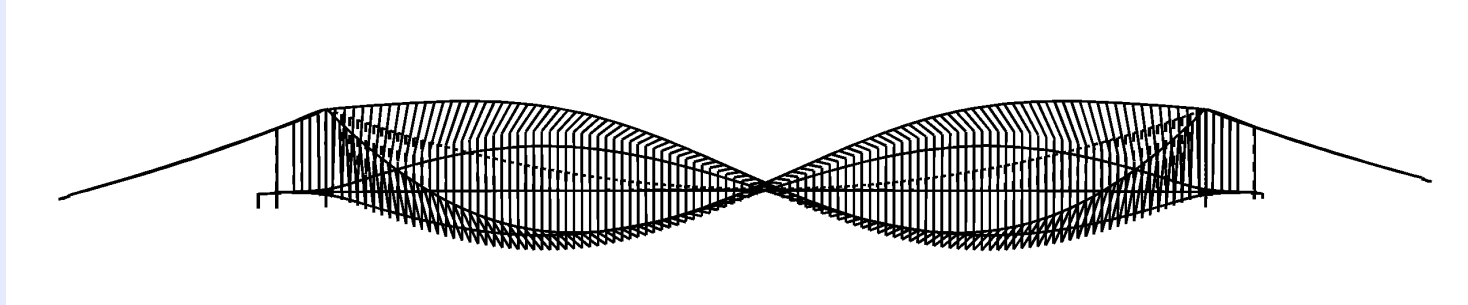
1<sup>st</sup> bending mode has **asymmetric** mode shape.  
 $f = 0.063\text{Hz}$ ,  $T = 15.8\text{sec}$ ,  $M_{eq} = 60.3\text{t/m}$



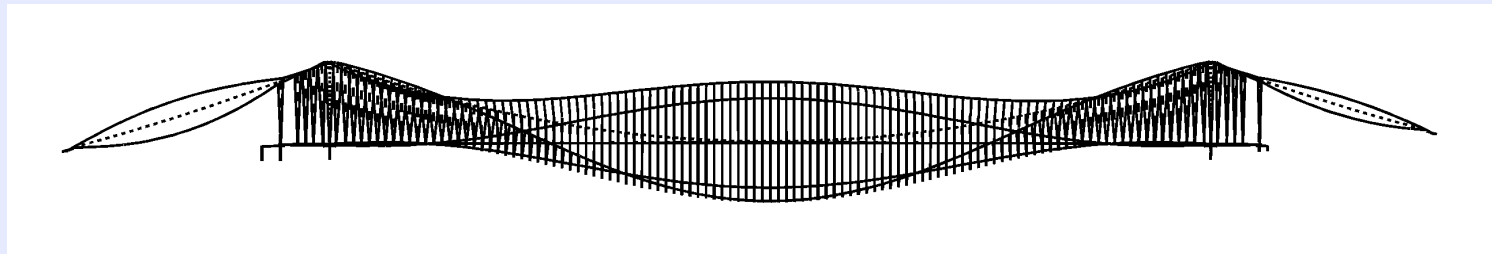
2<sup>nd</sup> bending mode has **symmetric** mode shape.  
 $f = 0.078\text{Hz}$ ,  $T = 12.8\text{sec}$ ,  $M_{eq} = 57.0\text{t/m}$

# Vibration Mode

- 1<sup>st</sup> & 2<sup>nd</sup> modes of torsional motion -



1<sup>st</sup> torsional mode has **asymmetric** mode shape.  
 $f = 0.090\text{Hz}$ ,  $T = 11.2\text{sec}$ ,  $I_{eq} = 32.421\text{tm}^2/\text{m}$

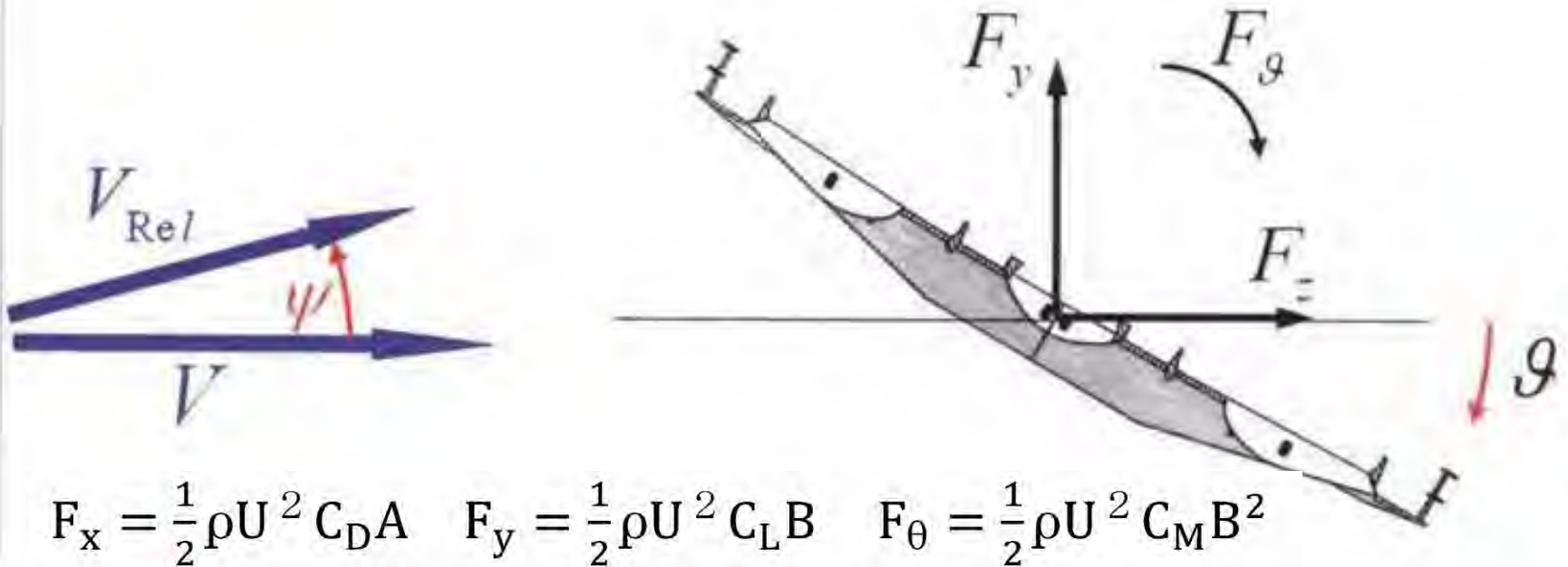


1<sup>st</sup> torsional mode has **symmetric** mode shape.  
 $f = 0.101\text{Hz}$ ,  $T = 9.9\text{sec}$ ,  $I_{eq} = 32.203\text{tm}^2/\text{m}$

# Some comments on Vibration Characteristics

- ◆ The vibration characteristics (natural freq. & vibration mode) is consistent with the results of Prof. Diana's research group.
- ◆ The 1<sup>st</sup> bending and torsional modes have asymmetrical mode shape.
- ◆ The predictive flutter mode will be asymmetrical mode. Therefore, in flutter analysis, the asymmetrical mode may be selected as the key vibration mode of coupled flutter.

# Static Aerodynamic Force Coefficients - Sign Convention -



$$F_x = \frac{1}{2} \rho U^2 C_D A \quad F_y = \frac{1}{2} \rho U^2 C_L B \quad F_\theta = \frac{1}{2} \rho U^2 C_M B^2$$

$F_i$  : aerodynamic force per unit length

$U$  : mean wind velocity,  $\rho$  : Air density

$A$  : projection area per unit length ( $m^2/m$ )

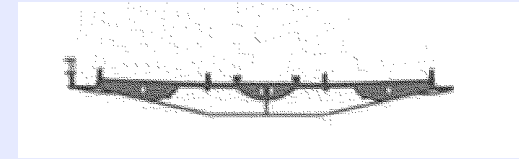
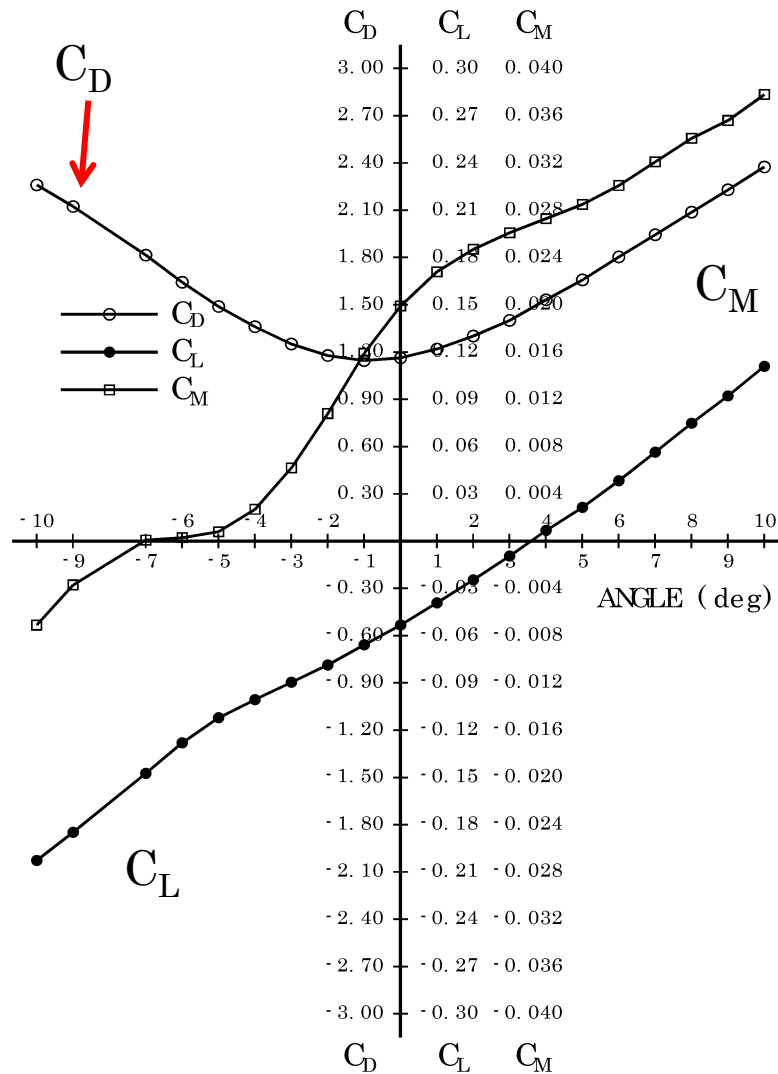
$B$  : bridge deck width (m)

$C_i$  ( $i = D, L, M$ ) : static aerodynamic force coefficient

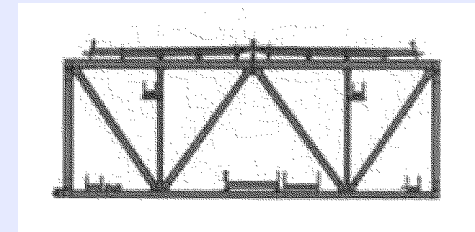
( $D$  : Drag,  $L$  : Lift,  $M$  : Moment)

# Static Aerodynamic Force Coefficients

## - Measured Data -



Messina Straits Bridge



Akashi-Kaikyo Bridge

	Messina	Akashi-Kaikyo
$C_D$	1.164	1.993
$C_L$	-0.053	0.0080
$dC_L/da$	0.765	1.446
$C_M$	0.020	-0.0046
$dC_M/da$	0.198	0.337

# Static Horizontal Deflection due to Wind of Full Aeroelastic Model at $U_p = 60\text{m/s}$



Maximum Deflection at Center Span

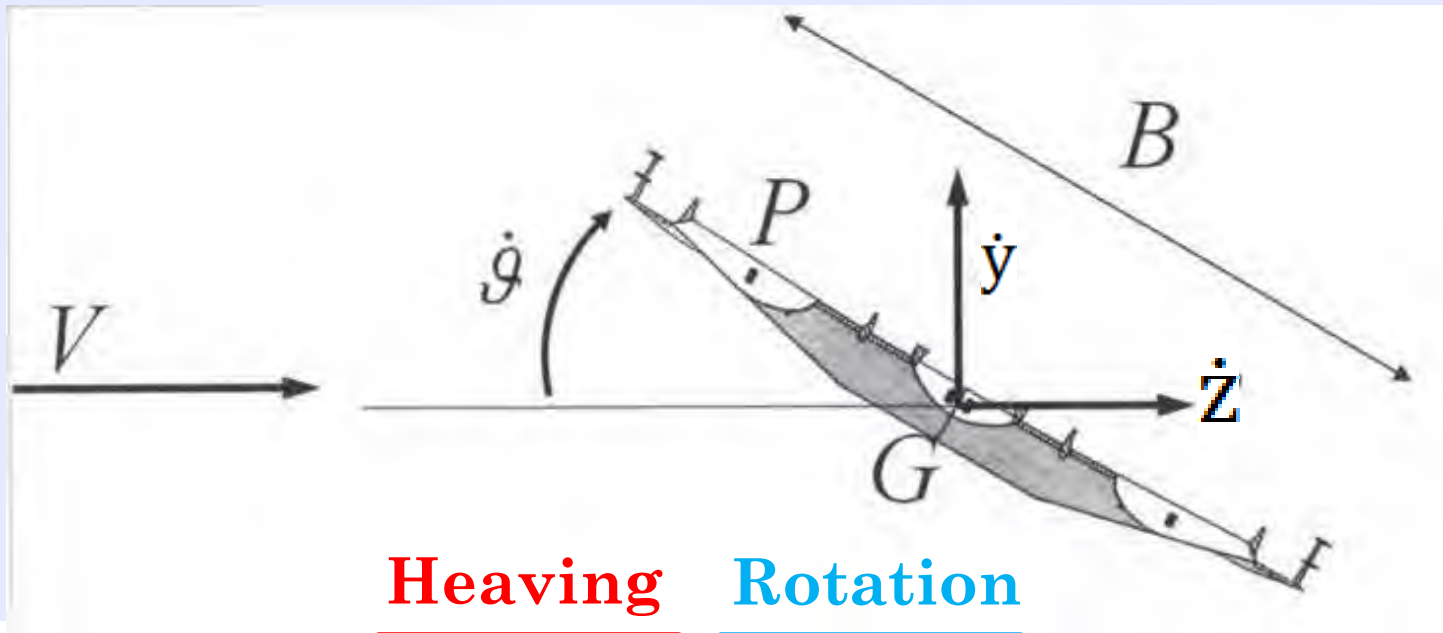
Messina Straits Bridge : around **10m**, Akashi Kaikyo Bridge : about **30m**

Reference from G. Diana : Messina Bridge Project – Technical Challenges -, 2006



# Flutter Derivatives of deck girder

## - Motion Induced Aerodynamic Force -

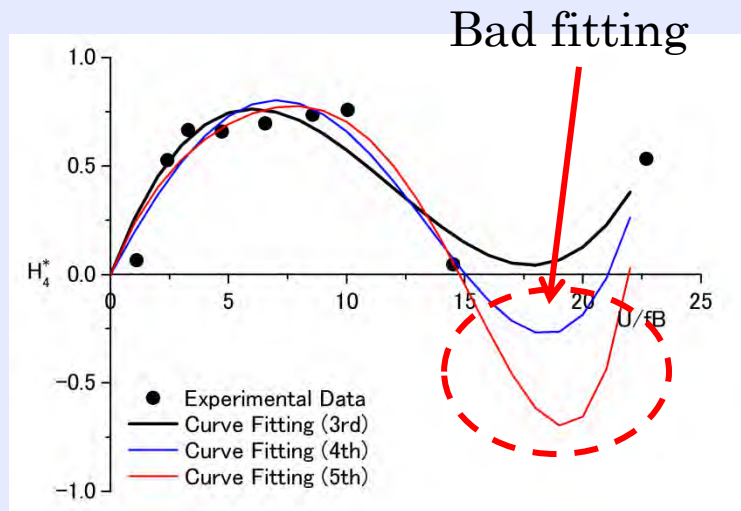
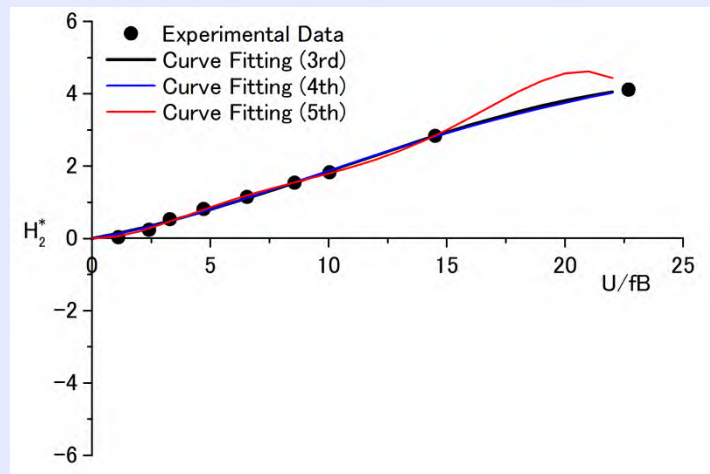
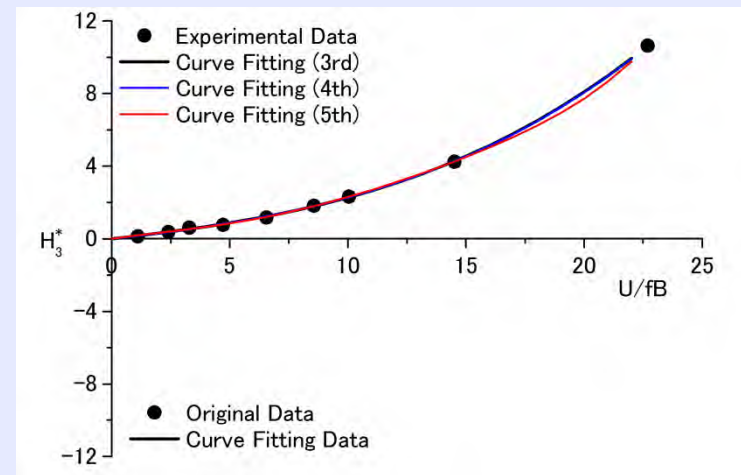
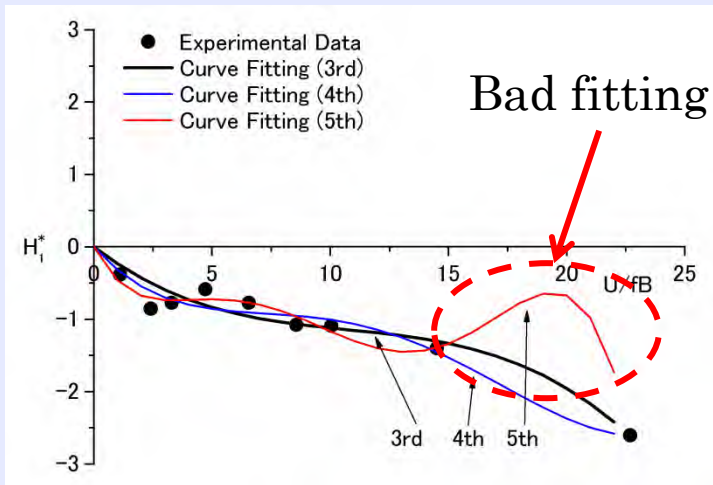


**Heaving**      **Rotation**

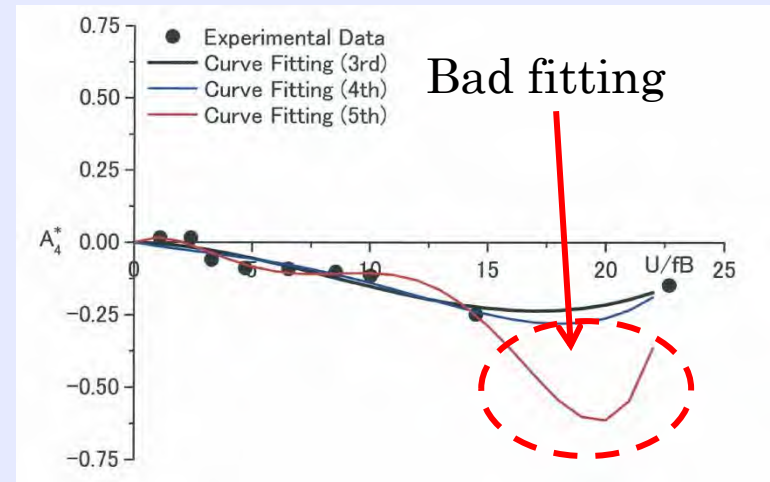
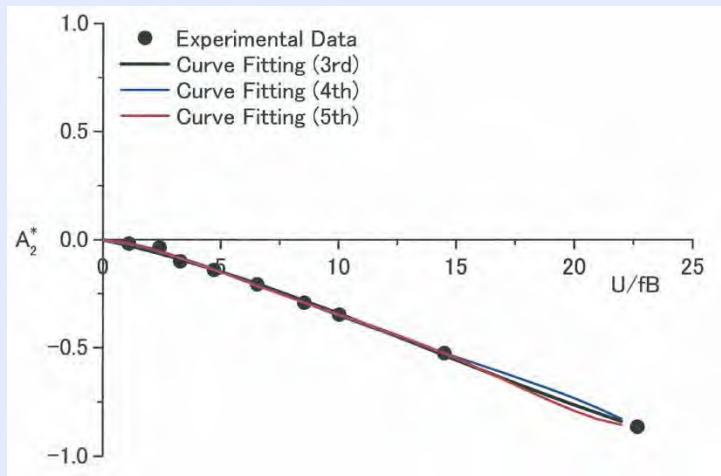
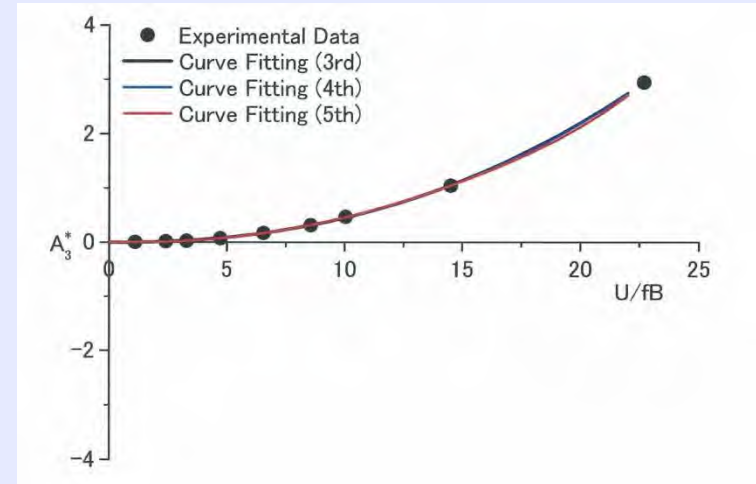
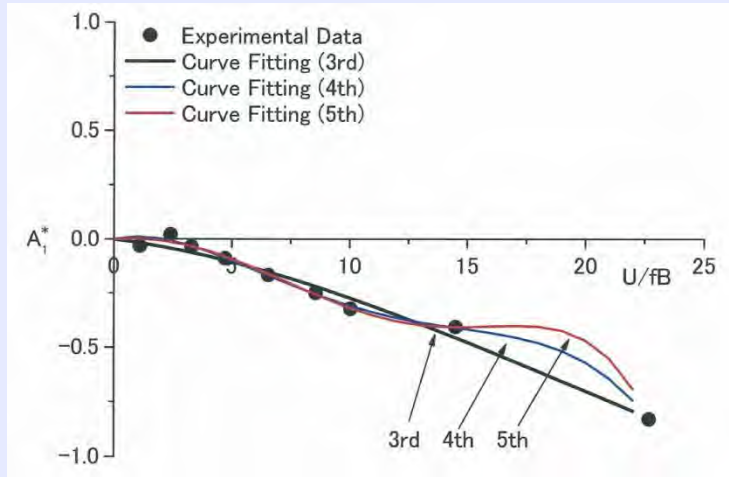
$$F_y = pBL \frac{1}{V^{*2}} \left\{ \boxed{(H_4^* + iH_1^*) \frac{y}{B}} + \boxed{(H_3^* + iH_2^*) \vartheta} \right\}, \quad V^* = U/fB$$

$$F_\vartheta = pB^2L \frac{1}{V^{*2}} \left\{ \boxed{(A_4^* + iA_1^*) \frac{y}{B}} + \boxed{(A_3^* + iA_2^*) \vartheta} \right\},$$

# Flutter Derivatives ( $H_i^*$ ) for Lift Force



# Flutter Derivatives( $A_i^*$ ) for moment force

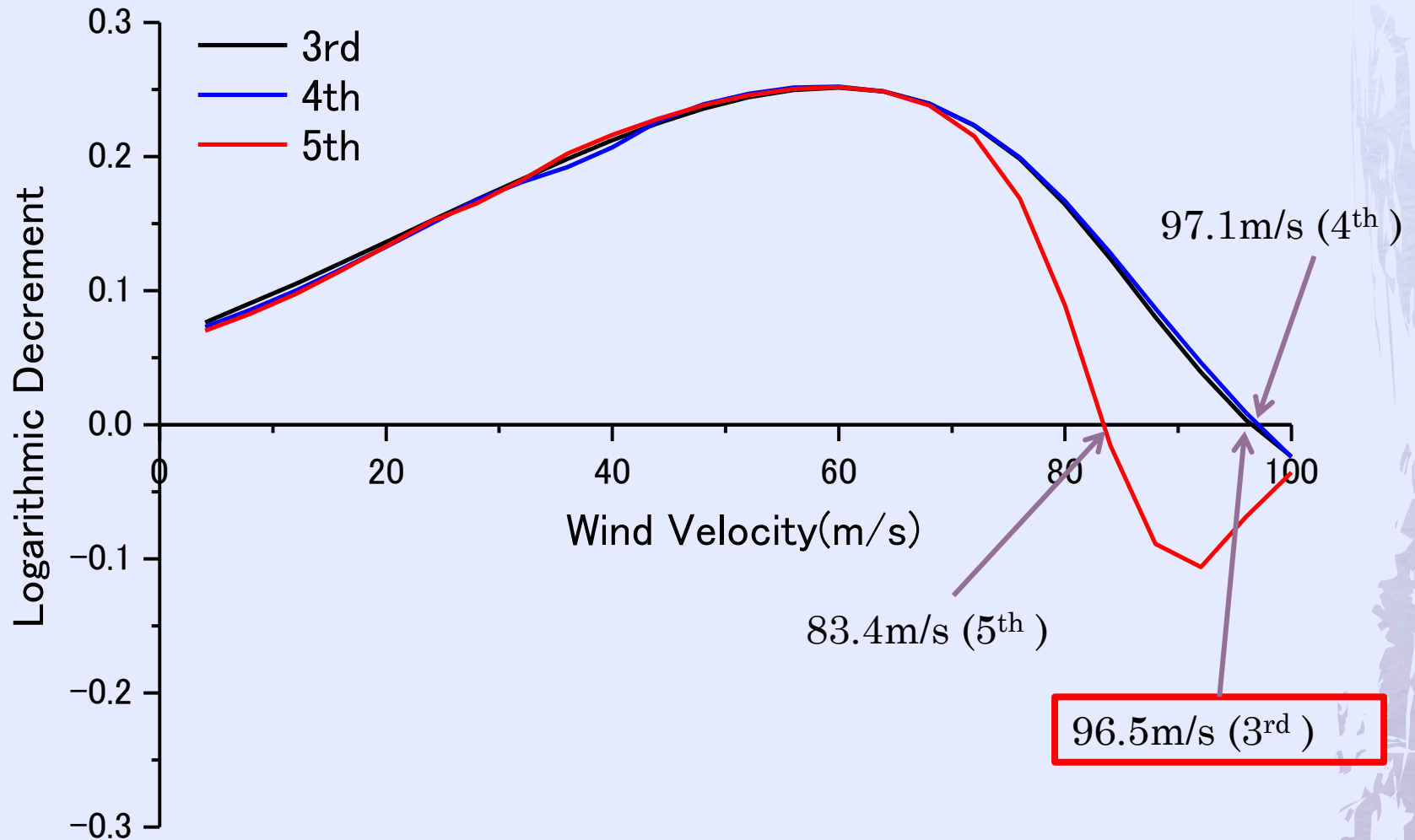


# Comparison of fitting curves for flutter derivatives by 2D flutter analysis

## - Vibration Characteristics of 2D Rigid Model-

Notation		Unit	Value
B	Bridge Deck Width	m	60
A	Projection Area per Unit Length	m	4.68
$f_h$	Natural Frequency of Vertical Motion	Hz	0.0634
$f_\theta$	Natural Frequency of Torsional Motion	Hz	0.0895
m	Mass per Unit length	t/m	60.2
I	Inertia Mass per Unit Length	tm <sup>2</sup> /m	3242
$\delta_h$	Structural Damping of Vertical Motion	—	0.0628
$\delta_\theta$	Structural Damping of Torsional Motion		(h = 1%)

# Comparison of fitting curves for flutter derivatives by 2D flutter analysis - V - $\delta$ Curve -



# Flutter Derivatives for 3D Flutter Analysis

		Motion					
		Sway		Vertical		Rotational	
		Vel.	Disp.	Vel.	Disp.	Vel.	Disp.
Force	Drag	$Q(P_1^*)$	-	$Q(P_0^*)$	-	$Q(P_3^*)$	-
	Lift	$Q(H_0^*)$	-	$M(H_1^*)$	$M(H_4^*)$	$M(H_2^*)$	$M(H_3^*)$
	Moment	$Q(H_0^*)$	-	$M(A_1^*)$	$M(A_4^*)$	$M(A_2^*)$	$M(A_3^*)$

$$P^*_{0i} = - (dC_D / d\alpha) / K_i = - C'_{Di} / K_i$$

$$P^*_{1i} = - 2C_{Di} / K_i, P^*_{2i} \doteq 0^3)$$

$$P^*_{3i} = (dC_{Di} / d\alpha) / K_i^2 = C'_{Di} / K_i^2$$

$$H^*_{0i} = - 2C_{Li} / K_i, A^*_{0i} = - 2C_{Mi} / K_i$$

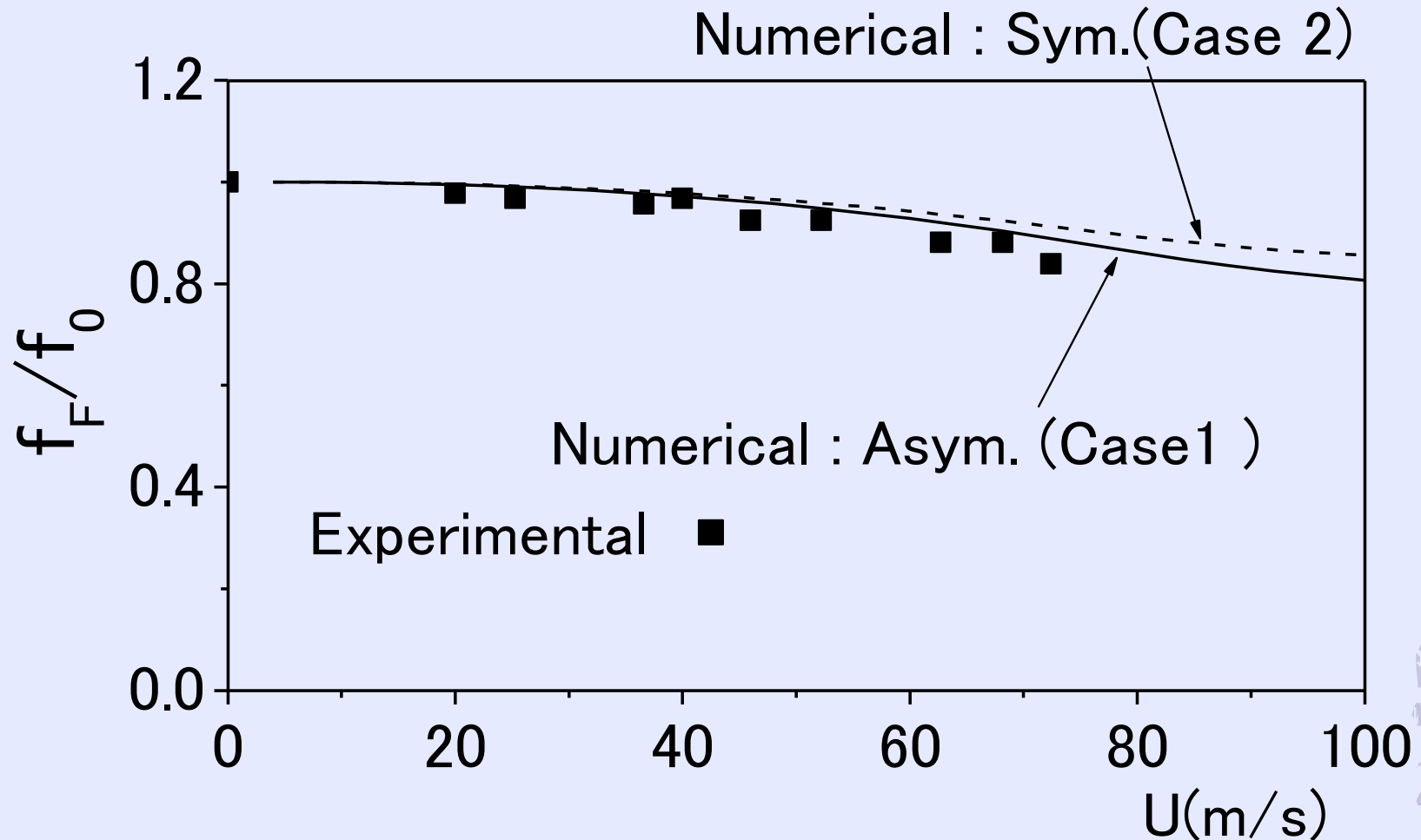
Measured values  
as the benchmark data

Calculated values by  
quasi steady theory

# Input Data of 3D Flutter Analysis

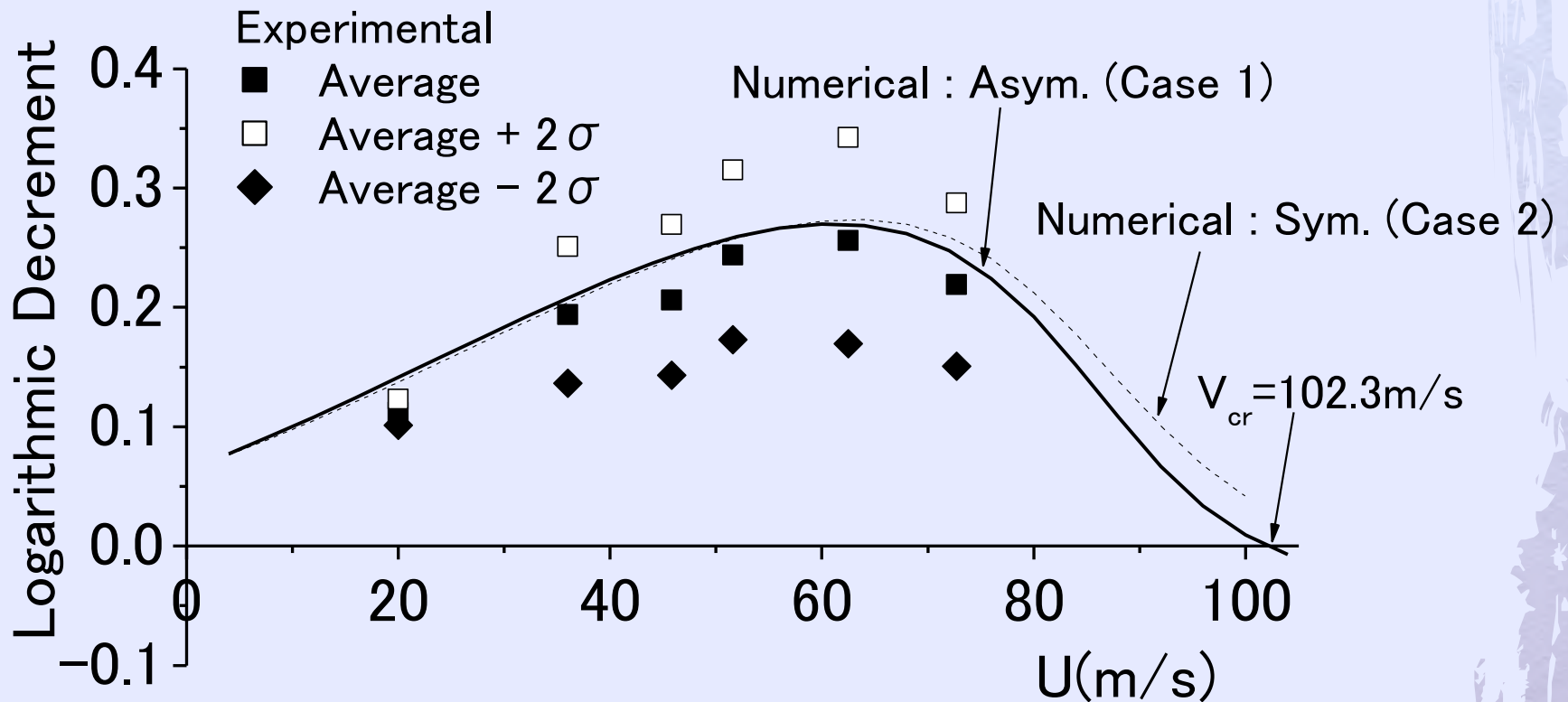
Static Aerodynamic Force	<ul style="list-style-type: none"><li>① Bridge Deck : Experimental data at <math>\alpha=0\text{deg}</math>.</li><li>② Cable : <math>C_D = 0.7</math></li><li>③ Hanger Cable : No consideration</li><li>④ Tower : <math>C_D = 1.8</math></li></ul>
Flutter Derivatives	<ul style="list-style-type: none"><li>① Bridge Deck Measured data for benchmark Calculated data by quasi steady theory</li><li>② Cable <math>H_1^*</math> was calculated by quasi steady theory.</li></ul>
Structural Damping	<ul style="list-style-type: none"><li>① Sway motion <math>\delta = 0.0251</math> (h = 0.4%)</li><li>② Vertical and rotational motions <math>\delta = 0.0628</math> (h = 1%)</li></ul> (* Measured values of aeroelastic model
Air Density	0.12 ( $\text{kg} \cdot \text{s}^2/\text{m}^4$ )

# Results of 3D Flutter Analysis - Mode Frequency Curve -



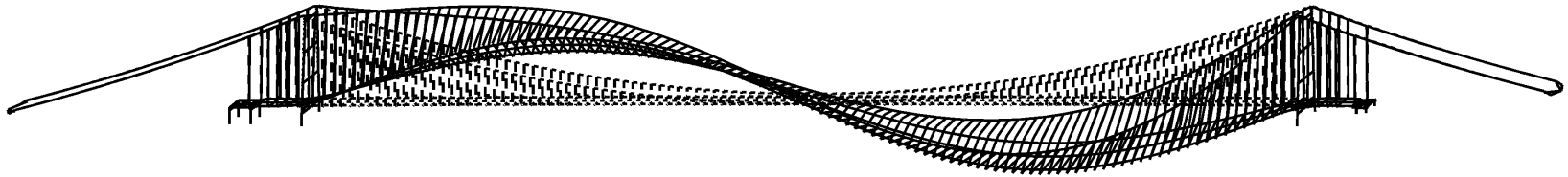


# Results of 3D Flutter Analysis - Structural Damping Curve -



# Results of 3D Flutter Analysis - Flutter Mode Shapes -

Wind Velocity = 104m/s



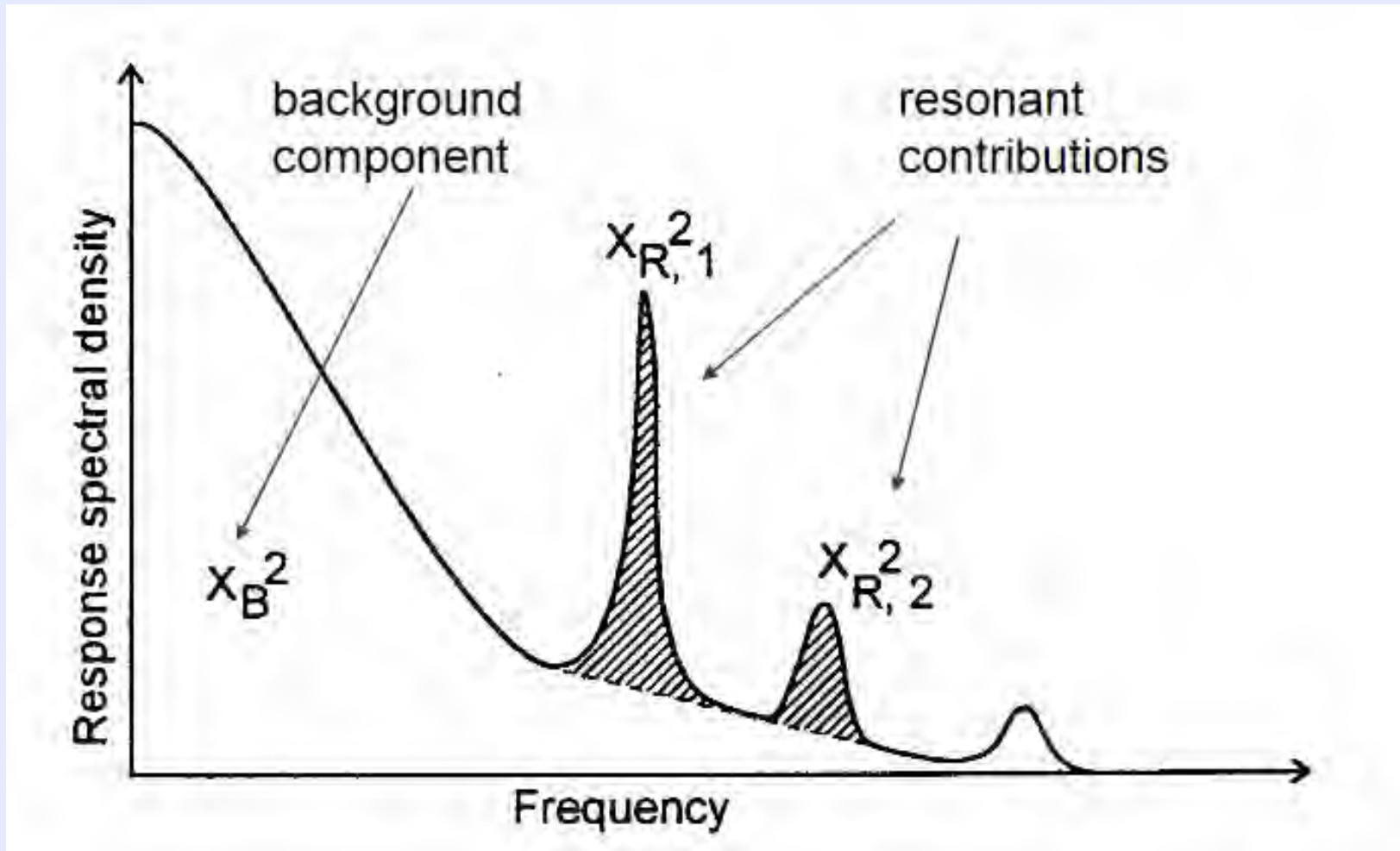
Frequency = 0.072Hz, Log. decrement = -0.00705

# Outline of Gust Response Analysis

- ◆ The gust response analysis is the **3-dimensional gust response analysis** of multi-degree of freedom system with 3-dimensional frame model.
- ◆ Buffeting forces of drag, lift and moment are formulated as **quasi-steady aerodynamic forces** with horizontal and vertical fluctuating wind velocities.
- ◆ The power spectral density functions of real buffeting force is also considered by **aerodynamic admittance functions**.
- ◆ Based on random vibration theory, **the integration of the power spectral density function of gust responses** gives variance of the gust response in the n-th mode as **resonant response**.
- ◆ In addition to resonance response, the quasi steady response (**background response**) is also calculated.
- ◆ **The root mean square response for the 50 modal responses** is composed by summation of variance of all modes.
- ◆ **The maximum expected responses** are calculated by multiplying the root mean square responses by **gust peak factor** defined by Davenport.

# Gust Response

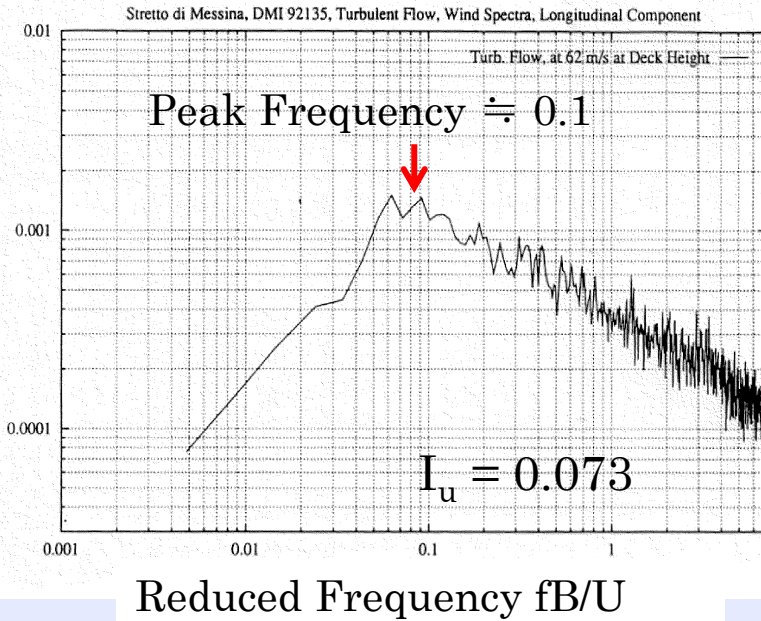
## - Resonant & Background Responses -



Reference from J.D.Holmes : Along Wind Response

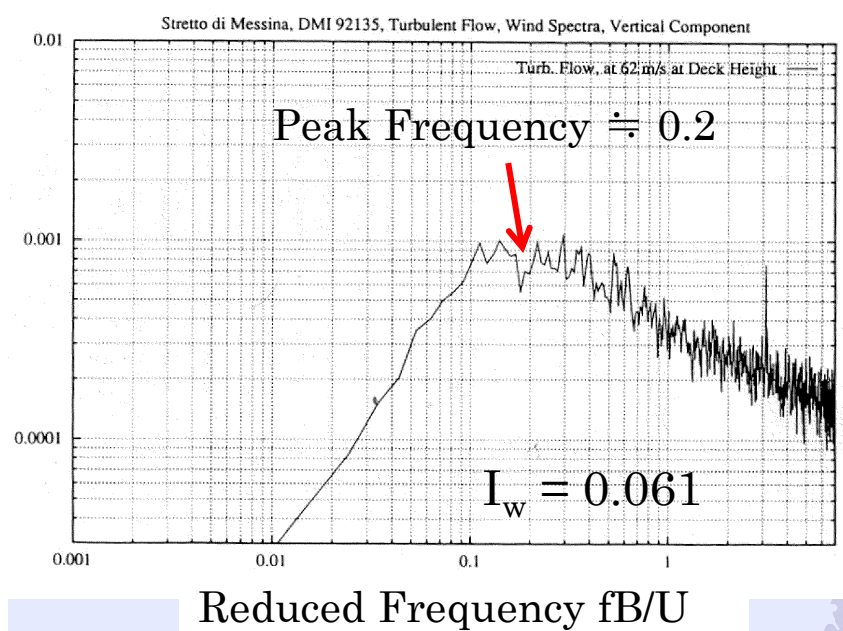
# Power Spectra of Wind Gust at Deck Height - Measured data in boundary layer flow -

Normalized Power Spectra  $fS_u/U^2$



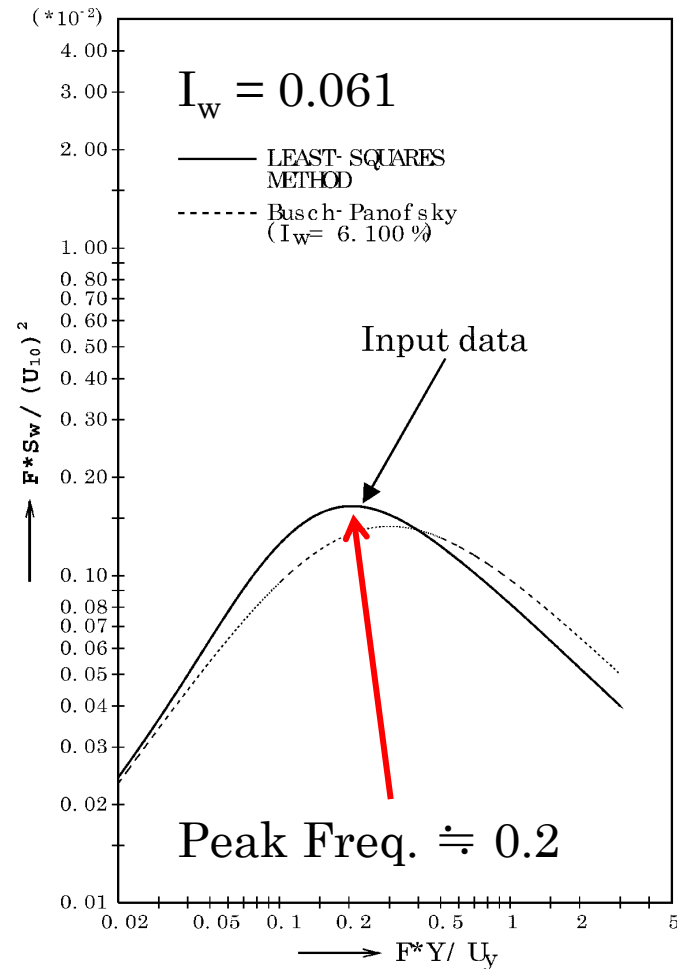
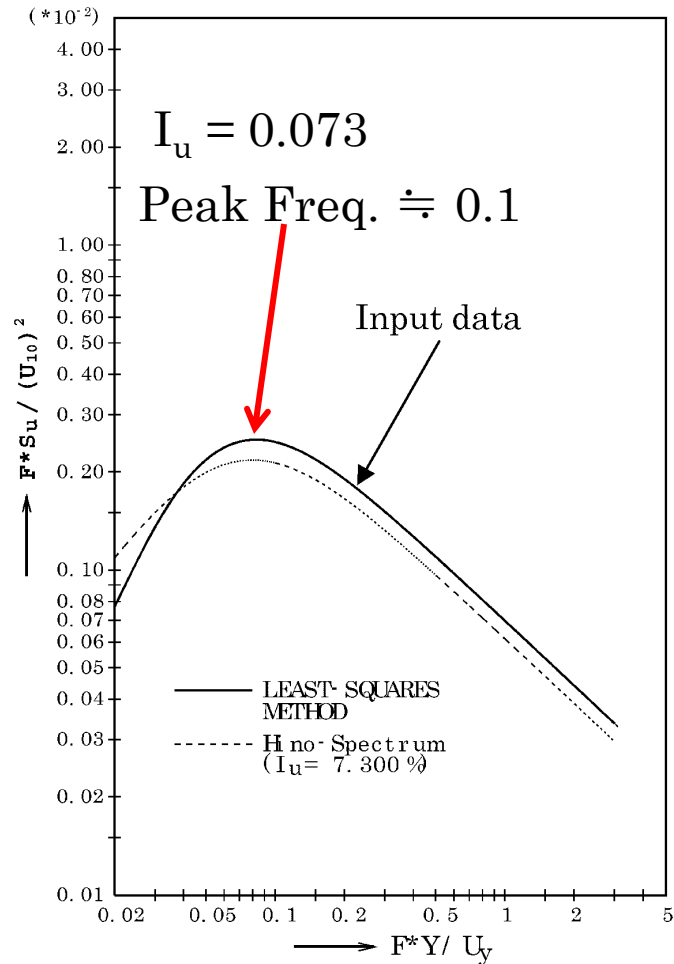
(a) Horizontal Comp.

Normalized Power Spectra  $fS_w/U^2$

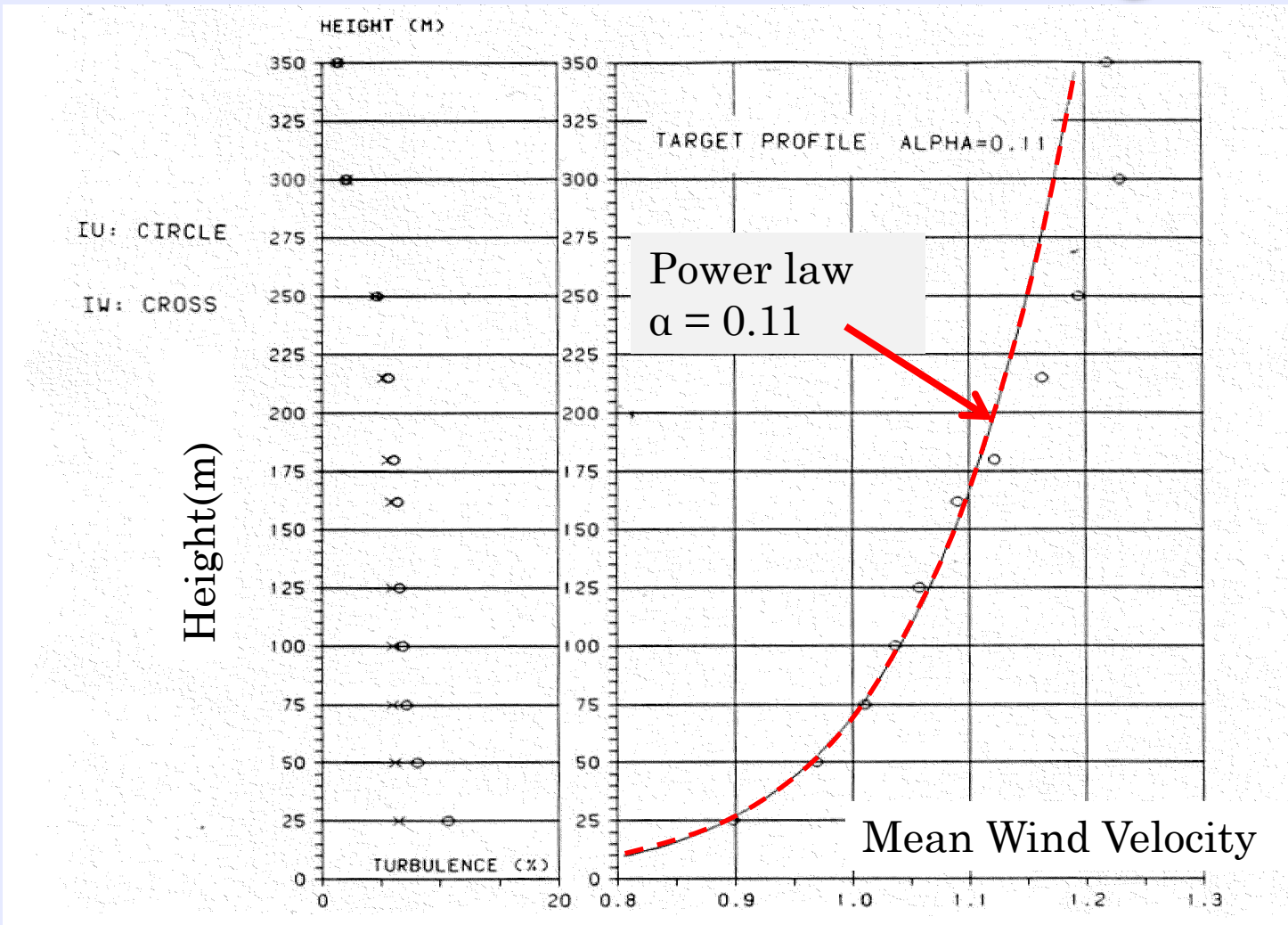


(b) Vertical Comp.

# Power Spectra of Wind Gust at Deck Height - Input data for gust response analysis-

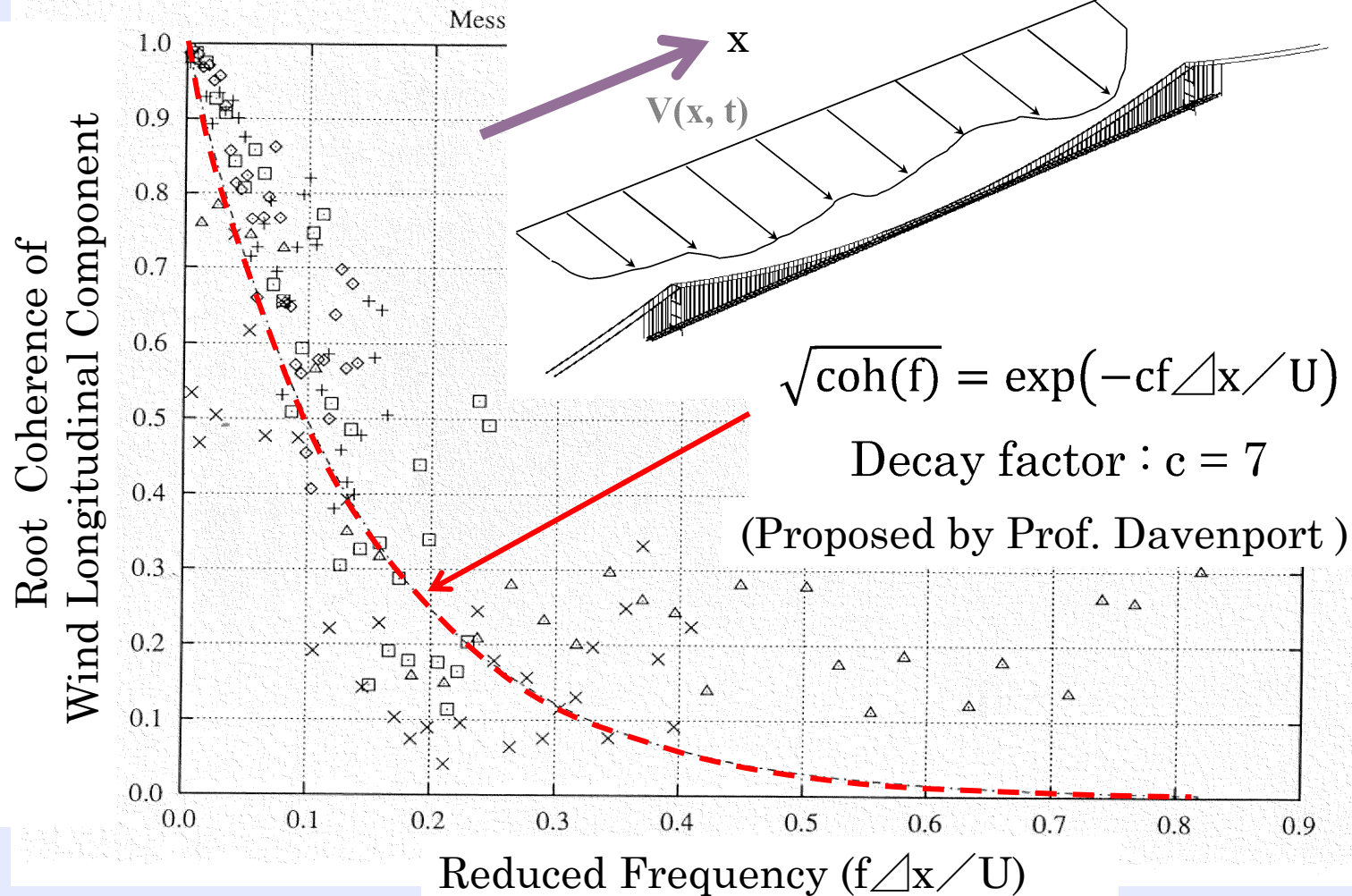


# Vertical Profile of wind velocity and turbulent intensity



Turbulent Intensity(%)

# Spatial Correlation of Horizontal Wind Gust





# Input Data of 3D Gust Response Analysis

Aerodynamic Admittance	Drag : Davenport Formula Lift and Moment : Sears Function
Spatial Correlation	Davenport Formula (Decay Factor : $k = 7$ )
Wind Power Spectra	The fitted wind power spectra to the measured wind power spectra in boundary layer turbulent flow
Wind Power	Power law of vertical profile $\alpha = 0.11$
Peak Factor	Davenport Formula ( $T = 600\text{sec}$ )

(\*) Input data of static and dynamic aerodynamic force, structural damping and air density are equal to the data of flutter analysis.

# Results of Gust Response Analysis

		NTI(①)	Diana(②)	Exp. (③)
Lateral(m)	Mean	9.91(1.00)	9.49( <b>0.96</b> )	8.36( <b>0.84</b> )
	RMS	1.81(1.00)	—	<u>0.28</u> ( <b>0.15</b> )
Vertical (1/2)(m)	RMS	0.44(1.00)	—	0.26( <b>0.59</b> )
Vertical (1/4)(m)	RMS	0.50(1.00)	0.43( <b>0.86</b> )	0.29( <b>0.58</b> )
Rotational (deg.)	Mean	0.64(1.00)	0.52( <b>0.81</b> )	0.40( <b>0.63</b> )
	RMS	0.29(1.00)	0.26( <b>0.90</b> )	0.17( <b>0.59</b> )

# Refinement of Gust Response Analysis

- ◆ For refinement of horizontal gust response
  - Recalculation of spatial correlation of horizontal wind gust
  - Modification of Davenport formula
- ◆ For refinement of vertical and rotational gust response
  - Use of the aerodynamic admittance function measured by Prof. Diana

# Spatial Correlation of Horizontal Wind Gust - for good fitting to experimental data -

## ① Davenport Formula

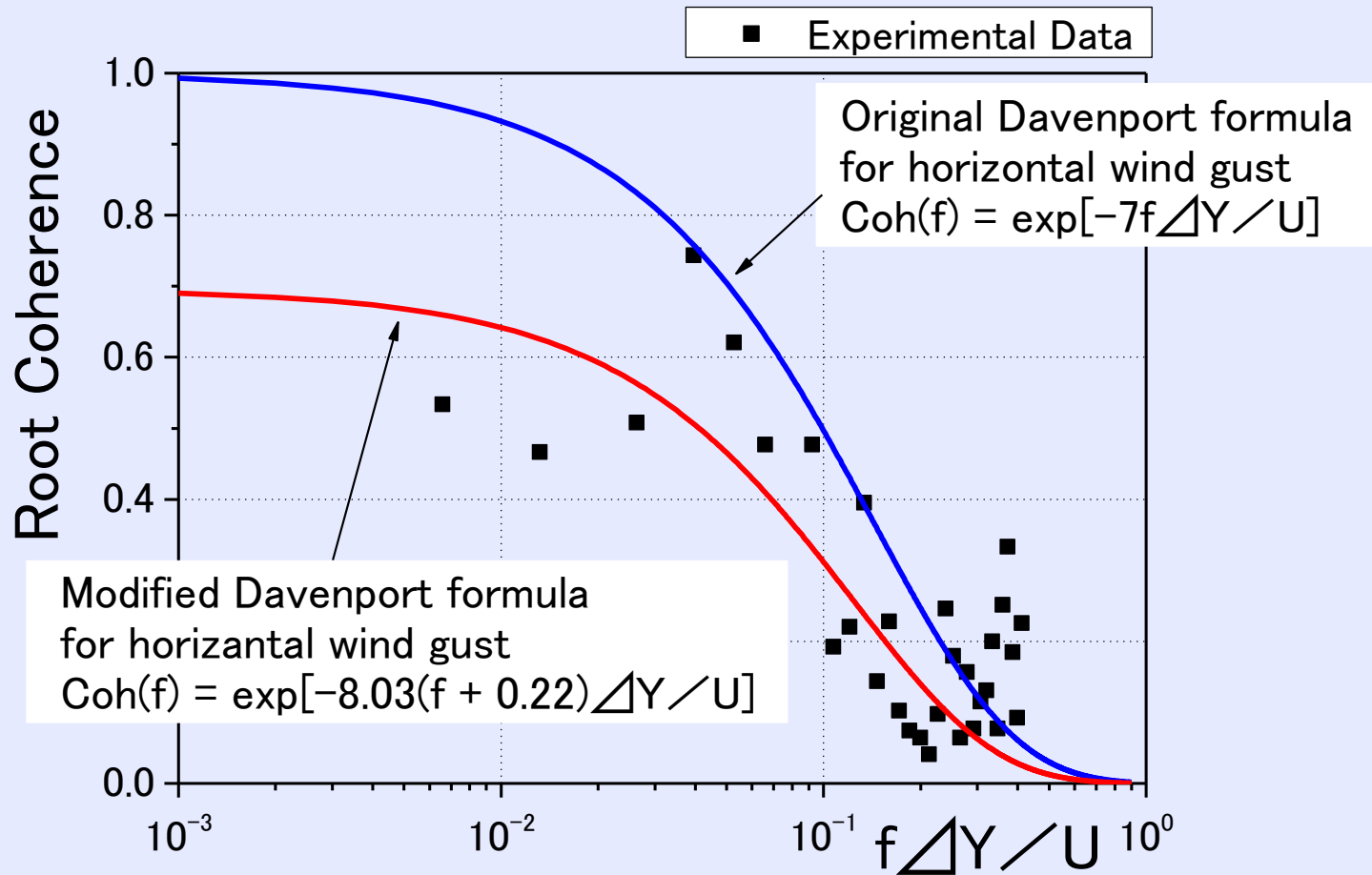
$$\sqrt{\text{coh}(f)} = \exp(-cf \Delta x / U)$$

## ② Modified Davenport Formula

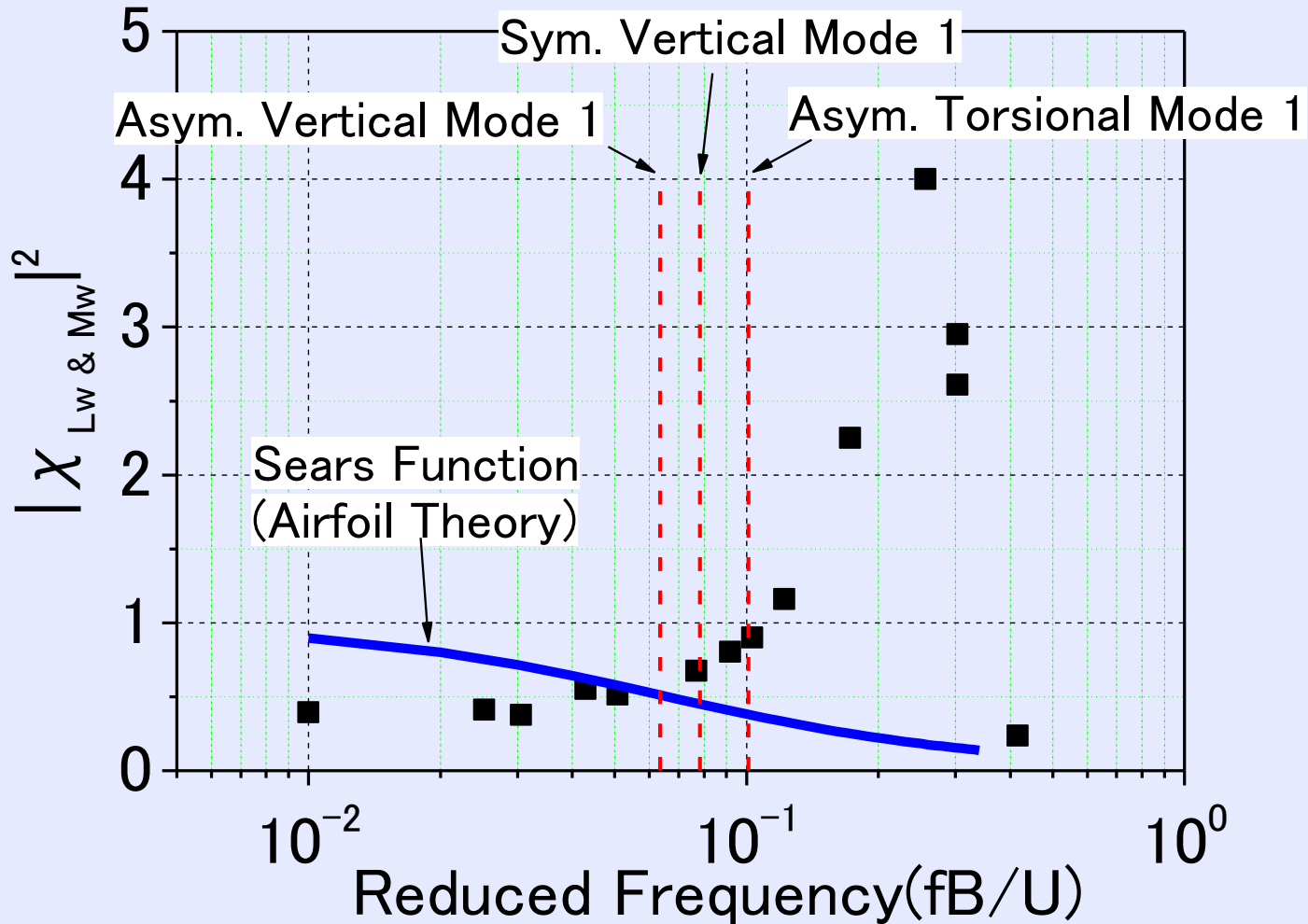
$$\sqrt{\text{coh}(f)} = \exp[-c(f + \mathbf{f}_0) \Delta x / U]$$

$f_0$  is identified as the fitting parameter to the experimental data.

# Refinement of spatial correlation of horizontal wind gust



# Use of the measured aerodynamic admittance function for lift and moment forces



# Results of Gust Response Analysis

		NTI(①)	Diana(②)	Exp. (③)
Lateral(m)	Mean	9.91(1.00)	9.49( <b>0.96</b> )	8.36( <b>0.84</b> )
	RMS	0.55(1.00)	—	<u>0.28</u> ( <b>0.51</b> )
Vertical (1/2)(m)	RMS	0.20(1.00)	—	0.26( <b>1.27</b> )
Vertical (1/4)(m)	RMS	0.21(1.00)	0.43( <b>0.86</b> )	0.29( <b>1.39</b> )
Rotational (deg.)	Mean	0.64(1.00)	0.52( <b>0.81</b> )	0.40( <b>0.63</b> )
	RMS	0.19(1.00)	0.26( <b>0.90</b> )	0.17( <b>0.89</b> )

# Conclusions

## - Natural Frequency Analysis -

- ◆ The natural frequency in this analysis agreed to the original results by Prof. Diana's research group within the about 10% error.
- ◆ The lowest modes of bending and torsional motions have asymmetric mode shapes.



# Conclusions

## - Flutter Analysis -

- ◆ The flutter onset velocity of 3D frame model was 102m/s.
- ◆ The analysis results on flutter frequency and logarithmic damping agree well to the experimental results. The flutter mode had asymmetrical mode shape.

# Conclusions

## - Gust Response Analysis -

- ◆ The analysis results agreed well to the numerical results by Prof. Diana's research group. However, the analysis results were smaller than the experimental RMS responses.
- ◆ Especially, the analysis result of sway motion was very smaller than the experimental RMS responses.
- ◆ The large errors on the RMS response of sway motion were thought to be due to the estimation errors of spatial correlation.

Continued on the following page

# Conclusions

## - Gust Response Analysis -

- ◆ For the refinement of gust response analysis, Davenport formula was modified to fit the experimental data of spatial correlation.
- ◆ The experimental data of the aerodynamic admittance functions for lift and moment forces was used.
- ◆ The RMS response of sway motion is better than the previous analysis.
- ◆ The RMS response of torsional motion agrees well to the experimental response. However, the RMS responses of bending motion were smaller than the experimental responses.